

Space-time Singularities - Current Status

R. V. Saraykar

Department of Mathematics, R T M Nagpur University
University Campus, Nagpur-440033, India
Email : ravindra.saraykar@gmail.com

Abstract

A brief review and discussion on recent developments in the theory of space-time singularities is presented here.

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Physical phenomena in the universe take place in the arena of space, and evolve in time. The known laws of physics describe and govern these happenings that occur in nature. But then what are space and time, what are their interconnections if any, and how the space and time themselves originate, or whether each of these is actually infinite and endless? These are some of the most profound questions that have exercised greatest of minds in science and philosophy over past many centuries. General relativity suggests not to view gravity as a 'force' in the usual sense, but describes it as the curvature and geometry of the 'space-time continuum', which is our universe. General relativity describes the interplay of the space-time curvature, and the matter within it that generates such a curvature, just as a metal ball placed on a rubber sheet curves it. It is a classical theory that governs the universe in its large scale structure. Given any physical system governed mainly by gravity, such as a large collection of galaxies, or a massive star that is close to the end of its evolution having burnt all its nuclear fuel, the Einstein equations govern the future evolution of such a system in time.

Thus we can ask the questions such as how the universe of galaxies that is continuously expanding will evolve in future, and whether it will continue to expand or will it start shrinking at some point in time in future. Or one could ask, what will be the final end point of evolution of a massive star that has started contracting under the force of its own gravity when its internal fuel is exhausted.

One of the most remarkable predictions of general relativity, developed during the 1960s and early 1970s has been that, dynamical evolution of matter fields in a space-time generically produces a space-time singularity. See references [1] - [6]:

Black holes and naked singularities are two possible outcomes of the collapse of a dying massive star. At the heart of each is a singularity—a wad of matter so dense that it requires new laws of physics to describe. Anything that hits the singularity gets destroyed. In a black hole, the singularity is “clothed” — that is, surrounded by a boundary called the event horizon that hides it. Nothing that falls through this surface can ever get back out.

A naked singularity has no such boundary. It is visible to outside observers, and objects that fall toward the singularity can in principle reverse course right up to the moment of impact. In the case of both of these, the densities, space-time curvatures and all other physical quantities blow up and grow arbitrarily large, and thus all known physical laws no longer hold there. In that sense, the singularity is the end (or beginning) of the space and time themselves.

The singularity theorems developed by Roger Penrose, Stephan Hawking and Robert Geroch show that the evolution of matter fields in a space-time generically yields such a singularity, provided reasonable physical conditions are satisfied . See references [1]- [3].

These conditions are the causality ensuring that you do not return to your own past, a suitable energy condition ensuring the positivity of energy density, and formation of what are called ‘trapped surfaces’ in a space-time that indicate and characterize that the gravitational field is sufficiently strong.

We outline here some aspects of the formation of these singularities. The space-time singularities develop in cosmology, where they signal the beginning of time, and in gravitational collapse of massive stars, which is an issue of great interest in gravitation physics today that has been investigated in much detail in recent years in the Einstein theory.

With the birth of a naked singularity in the numerical study of an inhomogeneous dust collapse, a series of papers were written to find serious counter examples to strong and weak Cosmic Censorship Conjectures, given by Penrose.

Weak cosmic censorship conjecture states that

“Generic asymptotically flat initial data have a maximal future development possessing a complete future null infinity.”

This means that if we disregard exceptional initial conditions, no singularities are observed from infinity. This conjecture does not suffice in so far as it allows for singularities which are locally visible although not accessible to observation from infinity. Such singularities may affect the unique predictability of the outcomes of observations by local observers.

The strong cosmic censorship conjecture forbids the formation of singularities of this type. It states that

“Generic asymptotically flat initial data have a maximal future development which is locally inextendible as a Lorentzian manifold in a continuous manner.”

This is equivalent to saying that a space-time is globally hyperbolic. i.e., maximal Cauchy development from given initial data is the whole space-time. This is guaranteed by Global Existence Theorems with a particular initial data. A number of collapse scenarios were studied in detail (cf., [3]). The matter fields studied include null radiation, inhomogeneous dust, perfect fluids, imperfect fluids, counter-rotating dust particles, self-gravitating massless scalar field, collisionless matter etc. In the case of collisionless matter, electrovac Gowdy space-times and space-times with compact locally homogeneous Cauchy surfaces, global existence theorems have been proved by Rein, Randall, Moncrief, Chrusciel and other researchers, thereby proving Strong cosmic censorship conjecture.

On the other hand, in the examples mentioned above, presence of strong curvature naked singularities have been proved (cf., [4]). Then there are issues of stability of these naked singularities, as well as issues of stability of black holes also. For example, very recently, Joshi and Malafarina ([7]) considered gravitational collapse of massive matter cloud described by Oppenheimer and Snyder, and examined its stability under the introduction of small tangential stresses. They proved that introduction of tangential pressure, however small, can qualitatively change the final fate of collapse from a black hole to a naked singularity (cf., [4]), for other definition of stability). This shows instability of black hole formation in collapse and throws important light on the nature of cosmic censorship hypothesis and its possible formulation.

On the other hand, there have been some interesting recent results related to Kerr geometry. The most general stationary vacuum black hole solution of Einstein's equations in a four dimensional asymptotically flat space-time is the Kerr geometry, characterized only by its mass M and angular momentum J . Solutions spinning below the Kerr bound $cJ/GM^2 \leq 1$ possess an event horizon and are known as Kerr black holes. Solutions spinning faster than the Kerr bound describe a “naked singularity”, where classical general relativity breaks down and (unknown) quantum gravity effects take over. All existing evidence indicates that Kerr black holes are perturbatively stable, while Kerr solutions with $cJ/GM^2 > 1$ are unstable. Thus naked singularities can not form from black hole instabilities.

Jacobson and Sotiriou ([8]) showed that if one considers an almost extremal black hole, non-spinning particles carrying enough angular momentum to create naked singularities are allowed to be captured. Thus black hole horizons can be destroyed and this represents violation of cosmic censorship conjecture in four dimensional asymptotically flat space-times. However this analysis neglects conservative and dissipative self-force effects, and both these effects may be important (cf., [10]).

Keeping this in view, Barausse, Cardoso and Khanna ([9]) proved that radiation reaction effects can prevent the formation of naked singularities only for some of the

orbits for non-spinning particles around almost extremal Kerr black holes identified by Jacobson and Sotiriou ([8]). However for all orbits capable of producing naked singularities, the conservative self-force is non-negligible and seems to prevent particles from being captured, thus saving the cosmic censorship conjecture [see the references in this paper (e.g. [9] for numerical and other work)].

Thus we see that there are arguments on both sides, supporting and violating cosmic censorship conjecture. The arguments depend on the nature of matter chosen, the nature of symmetries that solutions of Einstein field equations possess and the issue of stability.

Thus, while formulating a provable version of Cosmic censorship conjecture, one has to take into account all these issues and behaviour of different matter fields. Then there is an important issue of stability / genericity of naked singularities and black holes which is not yet satisfactorily resolved. Stability is to be examined in terms of small perturbations in the initial data from which the collapse evolves. This issue and the issue of genericity has been treated in details in a recent paper ([11]).

Other Open issues are :

- (1) Non-spherical collapse - not much is known.
- (2) Cylindrical collapse, which is devoid of singularities, to be taken seriously or not. In this case, trapped surfaces do not form (work of Apostolatos and Thorne, see [4]).
- (3) Proper definition of stability and genericity of a black hole and that of a naked singularity.

Finally, astro-physically speaking, naked singularities could reveal their presence to astronomers in several ways :

- (i) High-energy explosions that produce naked singularities would brighten and fade in a distinctive way.
- (ii) Certain classes of gamma-ray bursts remain unexplained; naked singularities might account for them.
- (iii) Naked singularities would bend the light of background stars differently than black holes do.
- (iv) If a suspected black hole is spinning faster than a certain rate (which depends on its mass), it must actually be a naked singularity. The planned Square Kilometer Array (SKA) radio telescope would have the requisite precision to tell.

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