Jour.Tensor Society **ISSN : 0974-5428**
Vol. 6 No. 2 (2012) 47-52
(S.I. Husain commemorative issue)

Some Conceptual Problems in GR

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Abstract

"I am not aware that relativity is at present regarded by physicists as a theory that may be believed or not, at will" Clemence, G.M., 1947 [Rev. Mod. Phys. 19, 361].

Keywords and phrases : Schwarzschild solution, Black holes, Gravitational waves and pulsars.
2000 AMS Subject Classification : 83C15, 83C57.

1. Introduction

I am happy to dedicate this brief article to the memory of the late Professor Izhar Husain. I had known him from my childhood. The usual place for running into him would be some meeting or lecture on relativity ..... my father, as a doyen in the field had great appreciation for him. I knew and admired him both as a researcher as well as a teacher. And it is as a teacher that I think of him while writing this article. As a teacher one learns from one's students, especially from their questions. I myself have appreciated this truism and what I am writing about here are the questions I received while teaching relativity and cosmology ..... questions to which I have not got satisfactory answers. I have aired them before (see references [1] and [2]).

The quote from the paper by Clemence who did so much to review the crucial experimental tests of general relativity worked at a time when the credibility of the subject was not universal. Not so now; but even so the questions raised here need to be satisfactorily answered. I should also mention that this is my personal 'list of difficulties' and may not coincide with the views of other workers in general relativity and cosmology. My list is:

1. The Schwarzschild Solution
2. Black Holes in Astrophysics

3. Gravitational Waves and Pulsars
4. Strong Principle of Equivalence in the Early Universe

I begin in that order.

2. The Schwarzschild Solution

This solution is supposed to describe the simplest gravitational scenario, viz., the gravitational field around a spherically symmetric point mass. The Newtonian problem involves solving the following equation:

$$\Box^2 \phi = -4\pi G \rho \tag{1}$$

Solving for $\rho = M\delta(r)$, where the delta function is in three dimensions, we get for $r > 0$,

$$d^2\phi/dr^2 + (2/r)d\phi dr = 0,$$

which integrates to

$$\phi = A + (B/r) \tag{2}$$

From boundary conditions at infinity, we get $A = 0$. How is B determined? By Green's theorem, the flux of $(d\phi/dr)$ across a sphere with centre at the origin is $4\pi GM$. This fixes B as GM. In other words, the point mass singularity determines the potential uniquely in a self contained manner.

In GR, this does not seem possible! We start with the Schwarzschild line element:

$$ds^2 = exp\,\nu\, dt^2 - exp\,\lambda\, dr^2 - r^2[d\theta^2 + \sin^2\theta d\phi^2] \tag{3}$$

and substitute in Einstein's equations for $r > 0$. We get the solution:

$$exp\,\nu = exp\,(-\lambda) = 1 - (D/r) \tag{4}$$

How do we determine D? At this stage an appeal is made to the Newtonian approximation, and taking the weak field limit at large r, we derive the value of D as 2GM. Unlike the Newtonian case, we have not used the information of the point source at the origin. Why not use the field equation for r approaching 0 ? Instead we have gone far away from the origin to derive the value of D.

Why not use the field equation for λ? This gives

$$exp\,(-\lambda) = 1 - 8\pi G \int_0^r R^2 \rho dR/r \tag{5}$$

We have a problem if we interpret the integral on the R.H.S. as $M/4\pi r$, because the element of proper 3- volume is not what is given in the integral but has to be multiplied by $\exp \lambda/2$ as evaluated at R. To do this properly we have to write after Iben [3] and Bondi [4] integrals which define the following relation:

Gravitational Mass = Nucleonic mass+ Internal energy+ Gravitational Potential Energy

For a more detailed discussion of the problem see: Petrov, A. N. and JVN, Ref. [5]. However, our original problem is still not answered! So we try another way, by considering the field equation

$$R = -\kappa T \tag{6}$$

which can be written in the Schwarzschild spacetime as

$$X'' + 2X'/r + 2X/r^2 = -8\pi GT \tag{7}$$

where $X = \exp(-\lambda) - 1$. This equation integrates and gives the accepted (Newtonian) answer, provided $T = M\delta(r)$. But the static solution requires us to have

$$T_0^0 = T_1^1 = 0. \tag{8}$$

Thus there is a contradiction! The moral is : Keep away from the singularity at $r = 0$!

3. Black Holes in Astrophysics

Black holes form from the gravitational collapse of massive stars. But for an external observer, whose Schwarzschild radial coordinate is constant they are never formed! The astrophysicist may argue that the collapsing object has become practically black in a finite time as measured by the external observer, and hence may be called a black hole for all practical purposes. This may be acceptable as a practical alternative. However, this alternative leads us to greater difficulty!

Since the laws of black hole physics require the horizon to have formed — which never happens for an external observer, can these laws have any physical relevance? The proofs of these laws are based on the existence of the horizon.

No Hair Theorem is another property whose realization in practice has not been proved. The statement of the theorem is: "A collapsing object of any irregular shape will lose all its moments /dynamical information / other physical characteristics except mass, charge and angular momentum as it approaches the state of a black hole." This is known as Price's theorem and is proved by Price for small perturbations from spherical symmetry. For finite departures from spherical symmetry the non-linearity of the problem makes the solution intractable. Why should one believe it in the most general case ? In the absence of a general proof, what is the status of the statement that the most general black hole has mass, charge and angular momentum only? Is this an article of faith with the GR community or have I missed a general demonstration of this result?

4. Gravitational Waves and Pulsars

Since the close studies of the binary pulsar, PSR 1913+16 it is argued that these observations confirm the predictions of general relativity. However, to me it seems that there is no prediction from general relativity in this case, which can be tested. Take the precession of the periastron of the binary orbit of two neutron stars of which one is a pulsar. The orbit does show a precession: but are we justified in comparing the observed value with the theoretical one if the latter is incorrect ? What is usually done is to reduce the binary orbit to a Newtonian centre of mass frame and then apply the relativistic result to this orbit, as is done for Mercury's orbit around the Sun. The following argument shows why this is wrong for the binary pulsar.

First of all, in the most general case the two body problem in general relativity has not yet been solved. The Sun-Mercury system is treated as a one-body problem because the mass of Mercury is very much smaller about 1.6×10^{-7} that of the Sun. Thus the Sun is treated as a single gravitating body whose mass determines the Schwarzschild spacetime geometry. Mercury is then treated as a test particle describing geodesic motion in that spacetime. This is a well justified approach and can be applied to any binary system in which the mass ratio is small compared to unity. But, *this approximation does not hold for the above pulsar.* There the mass ratio is close to unity. Reducing the system to a centre of mass frame is justified in Newtonian mechanics wherein the equations of motion are linear; but not in relativity where the equations are nonlinear.

For the same reason, one needs to be careful about accepting the conclusion that the gravitational radiation and shrinking of orbit confirm the predictions of general relativity. What has been done is to solve the orbital equations in the Newtonian system and equate the energy loss of the orbit as calculated in the Newtonian way to the linearized radiation formula from relativity. The observed shrinkage rate of the orbit is then compared with this calculation. I am not certain that the hybrid Newtonian + Relativistic approach used here is justified.

5. Strong Principle of Equivalence in the Early Universe

The justification of using standard flat space physics in a covariant way in the presence of gravity lies in the strong principle of equivalence (SPE). In the SPE, we reduce the coordinate system in a local region V to a locally inertial one, and then apply flat space physics to it. Normally this procedure requires the validity of the following kind:

$$L = \text{Radial size of } V << 1/\sqrt{R} \tag{9}$$

where, R is the typical curvature component of spacetime. This is the 'flat earth' approximation.

This condition becomes difficult to satisfy in the early universe as was shown by Padmanabhan and Vasanthi (see ref. [6]) They took an arbitrarily early epoch t and asked what is the size of a region at this epoch to which flat spacetime approximation is applicable. Since in the early Friedmann model, the spacetime curvature R is proportional to $1/t^2$, the radial size can be limited to (with $c = 1$):

$$L = \varepsilon \times t, \varepsilon << 1 \tag{10}$$

In general ε is expected to be small compared to unity, say between 10^{-6} and 10^{-3}. But to define temperature of this epoch we need statistical mechanics requiring a large enough number of relativistic particles in a region of this size. The particle number density, assuming that it is large enough to apply standard statistical mechanics is

$$n \sim [g/\pi^2]\{kT/h\}^3 \tag{11}$$

where g is the effective number of spin degrees of freedom. The other quantities have their usual meanings.

We now calculate how many particles are present in our locally flat region of size L. Using the time-temperature relationship in the early universe (see ref. [7]) we finally get this number as:

$$N \sim \varepsilon^3/(30\sqrt{g})[T_P/T]^3 \tag{12}$$

From this we see that at the GUT epoch the Planck temperature T_P is only 10^4 times the GUT value so that (11) gives N around unity for $\varepsilon \sim 10^{-3}$. For a lower ε it will be a fraction of unity! Surely statistical mechanics cannot be defined in such cases. This difficulty is not there at the epochs when primordial nucleosynthesis took place for in that case the ratio T_P/T is quite large and we get $N >> 1$.

Thus we find that if we wish to use flat spacetime physics we must ensure that SPE is valid at their epochs and this requirement puts the validity of most flat spacetime concepts under suspicion. In such cases one needs to formulate theories (such as statistical mechanics) ab-initio in a curved spacetime.

6. Conclusions

These ideas reflect the problematic aspects of general relativity, in particular under the visors used by the author. They are being aired here so that others may think about them and, if possible, find solutions to their paradoxical conclusions.

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