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Flow of an Unsteady Dusty Fluid Through a Channel having Triangular Cross-section in Frenet frame field System under varying Pulsatile Pressure Gradient

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(*Dedicated to Prof. K. S. Amur on his 80th birth year*)

Abstract

The geometry of laminar flow of an unsteady viscous liquid with uniform distribution of dust particles through a channel having triangular cross-section under the influence of time dependent pressure gradient has been considered. The intrinsic decomposition of flow equations are carried out in Frenet frame field system. Initially the fluid and dust particles are assumed to be at rest. The exact solutions for velocities of fluid and dust particles are obtained using variable separable and Laplace transform techniques. Further the skin friction at the boundary plates are also calculated, and the changes in the velocity profiles with s and n are shown graphically.

Key Words : Frenet frame field system; triangular cross-section, channel, laminar flow, dusty fluid; velocity of dust phase and fluid phase.
2000 AMS Subject Classification : 76T10, 76T15.

1. Introduction

Studies on the influence of the dust particles on viscous fluid flows are of great technical importance in the fields of fluidization, electrostatic precipitation, polymer technology, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil and in the engineering problems concerned with atmospheric fallout,

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dust collection, nuclear reactor cooling, powder technology, acoustics, sedimentation, performance of solid fuel rock nozzles, batch settling, rain erosion, guided missiles and paint spraying etc.

The study of the motion of dusty viscous fluids has recently attracted by many researchers, who has influenced by the publication of P.G.Saffman [14] investigations, which reveal the effect of stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Further, G. Radhakrishnama Charya [11] has studied about the pulsatile flow of a dusty fluid through a constricted channel. Michael et al. [9] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in the semi-infinite space above a rigid plane boundary. C.B.Singh et al. [15] have discussed the unsteady flow of an electrically conducting dusty viscous liquid through a channel. E.Rukmangadachari et al. [12] have illustrated the solutions of dusty viscous flow through a cylinder of rectangular cross-section under time dependent pressure gradient and also [13] investigated dusty viscous flow through a cylinder of triangular cross-section. Jagjit Pal Kaur et al. [7] have studied an unsteady porous channel flow of a conducting fluid with suspended particles. N.C.Ghosh et al. [16] have obtained the analytical solutions for the dusty visco-elastic fluid between two infinite parallel plates under the influence of time dependent pressure gradient, using appropriate boundary conditions.

Frenet frames are a central construction in modern differential geometry, in which structure is described with respect to an object of interest rather than with respect to external coordinate systems. Some researchers like Kanwal [8], Truesdell [17], Indrasena [6], Purushotham et al. [10], Bagewadi et al. [1],[2] have applied differential geometry techniques to study the fluid flow. Further, the authors [1], [2] have studied two-dimensional dusty fluid flow in Frenet frame field system, which is one of the moving frame. Recently Gireesha et al. [4],[5] have studied the flow of unsteady dusty fluid in different regions under varying time dependent pressure gradients. The present work deals with the study of flow of an unsteady dusty fluid through a channel having triangular cross-section under the influence of pulsatile pressure gradient in frenet frame field system. By considering the fluid and dust particles to be at rest initially, the analytical expressions are obtained for velocities of both fluid and dust particles. Further the skin friction at the boundary is calculated. The velocity profiles of both fluid and dust phase are shown graphically for different time t.

2. Equations of Motion

The equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are given by [14]:

For fluid phase

$$\nabla \cdot \overrightarrow{u} = 0, \qquad \text{(Continuity)} \tag{2.1}$$

$$\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u} \cdot \nabla)\overrightarrow{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \overrightarrow{u} + \frac{kN}{\rho}(\overrightarrow{v} - \overrightarrow{u}), \tag{2.2}$$

(Linear Momentum)

For dust phase

$$\nabla \cdot \overrightarrow{v} = 0, \qquad \text{(Continuity)} \tag{2.3}$$

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v} \cdot \nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}) \qquad \text{(Linear Momentum)} \tag{2.4}$$

We have following nomenclature:

$\overrightarrow{u}$−velocity of the fluid phase, $\overrightarrow{v}$−velocity of dust phase, ρ−density of the gas, p−pressure of the fluid, N−number density of dust particles, ν−kinematic viscosity, $k = 6\pi a\mu$−Stoke's resistance (drag coefficient), a−spherical radius of dust particle, m−mass of the dust particle, μ−the co-efficient of viscosity of fluid particles, t−time.

3. Franet frame field system

Let $\overrightarrow{s}, \overrightarrow{n}, \overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively as shown in the Figure-1.

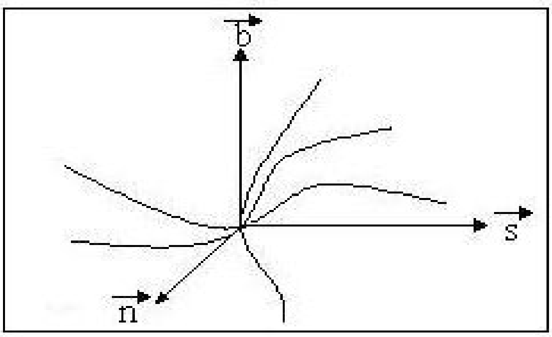

Figure 1. Frenet Frame Field System

Geometrical relations are given by Frenet formulae [3]

$$\begin{aligned}
&i) \quad \frac{\partial \vec{s}}{\partial s} = k_s \vec{n}, \ \frac{\partial \vec{n}}{\partial s} = \tau_s \vec{b} - k_s \vec{s}, \ \frac{\partial \vec{b}}{\partial s} = -\tau_s \vec{n} \\
&ii) \quad \frac{\partial \vec{n}}{\partial n} = k'_n \vec{s}, \ \frac{\partial \vec{b}}{\partial n} = -\sigma'_n \vec{s}, \ \frac{\partial \vec{s}}{\partial n} = \sigma'_n \vec{b} - k'_n \vec{n} \\
&iii) \quad \frac{\partial \vec{b}}{\partial b} = k''_b \vec{s}, \ \frac{\partial \vec{n}}{\partial b} = -\sigma''_b \vec{s}, \ \frac{\partial \vec{s}}{\partial b} = \sigma''_b \vec{n} - k''_b \vec{b} \\
&iv) \quad \nabla.\vec{s} = \theta_{ns} + \theta_{bs}; \ \nabla.\vec{n} = \theta_{bn} - k_s; \ \nabla.\vec{b} = \theta_{nb}
\end{aligned} \tag{3.1}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, tangential, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsions of the above curves and θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

4. Formulation and Solution of the Problem

Let us consider an unsteady flow of an incompressible viscous fluid with uniform distribution of dust particles through a channel having triangular cross section. The flow is only due to the influence of pulsatile pressure gradient. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size.

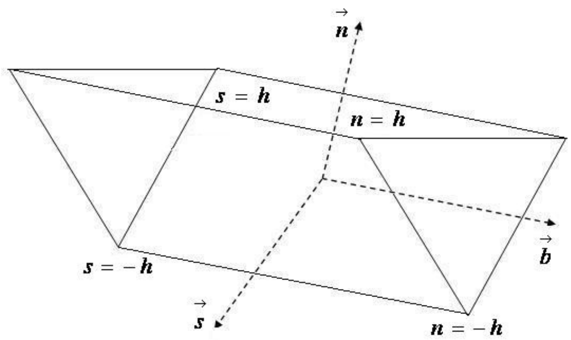

Figure 2. Geometry of the Flow.

The number density of the dust particles is taken as a constant throughout the flow. As Figure-2 shows, the axis of the channel is along binormal direction and the velocity components of both fluid and dust particles are respectively given by:

$$\vec{u} = u_b \vec{b}, \quad \vec{v} = v_b \vec{b}, \tag{4.1}$$

where (u_s, u_n, u_b) and (v_s, v_n, v_b) are velocity components of fluid and dust particles respectively.

By virtue of system of equations (3.1) the intrinsic decomposition of equations (2.2) and (2.4) using equation (4.1) give the following forms:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \nu\left(\tau_s k_s u_b - 2\sigma_n' \frac{\partial u_b}{\partial n}\right) \tag{4.2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial n} + \nu\left(\sigma_n' k_n' u_b - 2\tau_s \frac{\partial u_b}{\partial s}\right) \tag{4.3}$$

$$\frac{\partial u_b}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial b} + \nu\left[\frac{\partial^2 u_b}{\partial s^2} + \frac{\partial^2 u_b}{\partial n^2} - C_r u_b\right] + \frac{kN}{\rho}(v_b - u_b) \tag{4.4}$$

$$\frac{\partial v_b}{\partial t} = \frac{k}{m}(u_b - v_b) \tag{4.5}$$

$$v_b^2 k_b'' = 0 \tag{4.6}$$

where $C_r = (\tau_s^2 + \sigma_n'^2 + k''^2{}_b)$ is called curvature number [2].

From equation (4.6) we see that $v_b^2 k_b'' = 0$ which implies either $v_b = 0$ or $k_b'' = 0$. The choice of $v_b = 0$ is impossible, since if it happens, then $u_b = 0$, which shows that the flow doesn't exist. Hence $k_b'' = 0$, it suggests that the curvature of the streamline along binormal direction is zero. Thus no radial flow exists.

Since we have assumed that the pulsatile pressure gradient to be impressed on the system for $t > 0$, we can write

$$-\frac{1}{\rho}\frac{\partial p}{\partial b} = c_1 + c_2 \cos t \tag{4.7}$$

where c_1 and c_2 are constants.

Equation (4.4) and (4.5) are to be solved subject to the initial and boundary conditions:

$$\left\{\begin{array}{ll} \text{Initial condition;} & \text{at } t = 0;\ u_b = 0, v_b = 0 \\ \text{Boundary condition;} & \text{for } t > 0;\ u_b = 0 \ \text{ at } \ s = h \ \ \& \ \ s = -h, \\ & \text{and} \qquad u_b = 0 \ \text{ at } \ n = h \ \ \& \ \ n = -h \end{array}\right\} \tag{4.8}$$

Let U_b and V_b are given by

$$U_b = \int_0^\infty e^{-xt} u_b dt \quad \text{and} \quad V_b = \int_0^\infty e^{-xt} v_b dt \tag{4.9}$$

denote the Laplace transforms of u_b and v_b respectively.

Using relations (4.7) and (4.9) in equations (4.4), (4.5) and (4.8) one can obtain the following:

$$xU_b = \frac{c_1}{x} + \frac{c_2 x}{(x^2+1)} + \nu\left(\frac{\partial^2 U_b}{\partial s^2} + \frac{\partial^2 U_b}{\partial n^2} - C_r U_b\right) + \frac{l}{\tau}(V_b - U_b) \quad (4.10)$$

$$V_b = \frac{U_b}{(1+x\tau)} \quad (4.11)$$

$$U_b = 0 \text{ at } s = h \;\&\; s = -h \text{ and } U_b = 0 \text{ at } n = h \;\&\; n = -h, \quad (4.12)$$

where $l = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$.

From equations (4.10) and (4.11) we obtain, the following equation

$$\frac{\partial^2 U_b}{\partial s^2} + \frac{\partial^2 U_b}{\partial n^2} - Q^2 U_b + R = 0 \quad (4.13)$$

where

$$Q^2 = \left(C_r + \frac{x}{\nu} + \frac{xl}{\nu(1+x\tau)}\right) \; and \quad R = \frac{1}{\nu}\left[\frac{c_1}{x} + \frac{c_2 x}{x^2+1}\right]$$

To solve equation (4.10) we assume the solution in the following form [18]

$$U_b(s,n) = w_1(s,n) + w_2(s) \quad (4.14)$$

Substitution of $U_b(s,n)$ in equation (4.13) yields

$$\frac{\partial^2 w_1}{\partial s^2} + \frac{\partial^2 w_2}{\partial s^2} + \frac{\partial^2 w_1}{\partial n^2} - Q^2(w_1 + w_2) + R = 0$$

so that if w_2 satisfies

$$\frac{\partial^2 w_2}{\partial s^2} - Q^2 w_2 + R = 0$$

then

$$\frac{\partial^2 w_1}{\partial s^2} + \frac{\partial^2 w_1}{\partial n^2} - Q^2 w_1 = 0 \quad (4.15)$$

In similar manner if $U_b(s,n)$ is inserted in no slip boundary conditions, one can obtain

$$\left\{\begin{array}{l} U_b(h,n) = w_1(h,n) + w_2(h) = 0, U_b(-h,n) = w_1(-h,n) + w_2(-h) = 0, \\ U_b(s,h) = w_1(s,h) + w_2(s) = 0, \;\; U_b(s,-h) = w_1(s,-h) + w_2(s) = 0 \end{array}\right\}$$

By solving the problem

$$\frac{\partial^2 w_2}{\partial s^2} - Q^2 w_2 + R = 0, \quad w_2(h) = 0, \quad w_2(-h) = 0$$

we obtain the solution in the form

$$w_2(s) = \frac{R}{Q^2}\left(\frac{\cosh(Qh) - \cos(Qs)}{\cos(Qh)}\right) \quad (4.16)$$

Using variable separable method, the solution of the problem (4.15) with the conditions

$$w_1(h,n)=0, \quad w_1(-h,n)=0, \quad w_1(s,h)=-w_2(s), \quad w_1(s,-h)=-w_2(s)$$

is obtained in the form

$$w_1(s,n) = \sum_{r_1=0}^{\infty} \sin\left(\frac{r_1\pi}{h}s\right)\left(c_{r_1}e^{An}+D_{r_1}e^{-An}\right) \qquad (4.17)$$

where $A=\sqrt{\frac{Q^2h^2+r_1^2\pi^2}{h^2}}$

Now by substituting (4.16) and (4.17) in (4.14) we have

$$\begin{aligned} U_b(s,n) &= \frac{R}{Q^2}\left(\frac{\cosh(Qh)-\cosh(Qs)}{\cosh(Qh)}\right) \\ &+\frac{2R}{Q^2}\sum_{r_1=0}^{\infty}\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{(-1)^{r_1}Q^2}{A^2r_1\pi}+\frac{r_1\pi}{A^2h^2\cosh(Qh)}-\frac{1}{r_1\pi}\right\}\frac{\cosh(An)}{\cosh(Ah)}. \end{aligned}$$

Using U_b in equation (4.11) one can see that

$$\begin{aligned} V_b(s,n) &= \frac{R}{Q^2(1+x\tau)}\left(\frac{\cosh(Qh)-\cosh(Qs)}{\cosh(Qh)}\right)+\frac{2R}{Q^2(1+x\tau)}\sum_{r_1=0}^{\infty}\sin\left(\frac{r_1\pi}{h}s\right) \\ &\times\left\{\frac{(-1)^{r_1}Q^2}{A^2r_1\pi}+\frac{r_1\pi}{A^2h^2\cosh(Qh)}-\frac{1}{r_1\pi}\right\}\frac{\cosh(An)}{\cosh(Ah)} \end{aligned}$$

By taking inverse Laplace transformation to U_b and V_b, we obtain u_b and v_b as follows:

$$\begin{aligned} u_b(s,n,t) &= \frac{c_1}{\nu X^2}\left[\frac{cosh(Xh)-cosh(Xs)}{cosh(Xh)}\right]+\frac{c_2}{\nu}\left[\frac{k_1\cos t+k_2\sin t}{(y_1^2+z_1^2)(C^2+D^2)}\right] \\ &+\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2h}s\right]\left[\frac{e^{x_3t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\right. \\ &\times\left.\frac{c_1(x_3^2+1)+c_2x_3^2}{x_1(x_3^2+1)}+\frac{e^{x_4t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_2(x_4^2+1)}\right] \\ &+\frac{2}{\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu Y^2}\frac{\cosh(Yn)}{\cosh(Yh)}+\frac{c_2}{\nu}\frac{k_3\cos t+k_4\sin t}{(s_1^2+t_1^2)(G^2+H^2)}\right. \\ &+\frac{1}{\nu(x_5-x_6)}\left[\frac{e^{x_5t}c_1(x_5^2+1)+c_2x_5^2}{x_5(x_5^2+1)}-\frac{e^{x_6t}c_1(x_6^2+1)+c_2x_6^2}{x_6(x_6^2+1)}\right] \end{aligned}$$

$$- \frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]\left[\frac{e^{x_7t}(1+x_7\tau)^2}{[l+(1+x_7\tau)^2]}\right.$$

$$\left.\left.\times\frac{c_1(x_7^2+1)+c_2x_7^2}{x_7(x_7^2+1)}+\frac{e^{x_8t}(1+x_8\tau)^2}{[l+(1+x_8\tau)^2]}\frac{c_1(x_8^2+1)+c_2x_8^2}{x_8(x_8^2+1)}\right]\right\}$$

$$+\frac{2\pi}{h^2}\sum_{r_1=0}^{\infty}r_1\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu X^2Y^2}\frac{\cosh(Yn)}{\cosh(Xh)\cosh(Yh)}\right.$$

$$+\frac{c_2}{\nu}\frac{L_5\cos t+L_6\sin t}{(y_1^2+z_1^2)(s_1^2+t_1^2)(G^2+H^2)(C^2+D^2)}+\frac{1}{\nu(x_1-x_2)}$$

$$\times\left[\frac{e^{x_1t}c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}-\frac{e^{x_2t}c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\frac{\cosh(\alpha n)}{\alpha^2\cosh(\alpha h)}$$

$$-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cosh(\beta n)}{\beta^2\cosh(\beta h)}\left[\frac{e^{x_3t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_1(x_3^2+1)}\right.$$

$$\left.+\frac{e^{x_4t}(1+x_4\tau)^2}{[l+(1+x_4tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]+\frac{(-1)^{r_1}}{\nu\Gamma^2(x_5-x_6)}$$

$$\times\left[\frac{e^{x_5t}c_1(x_5^2+1)+c_2x_5^2}{x_5(x_5^2+1)}-\frac{e^{x_6t}c_1(x_6^2+1)+c_2x_6^2}{x_6(x_6^2+1)}\right]-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}$$

$$\times\frac{\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]}{\alpha_2\cos(\alpha_1h)}\left[\frac{e^{x_7t}(1+x_7\tau)^2}{[l+(1+x_7\tau)^2]}\frac{c_1(x_7^2+1)+c_2x_7^2}{x_7(x_7^2+1)}+\frac{e^{x_8t}(1+x_8\tau)^2}{[l+(1+x_8\tau)^2]}\right.$$

$$\left.\left.\times\frac{c_1(x_8^2+1)+c_2x_8^2}{x_8(x_8^2+1)}\right]\right\}-\frac{2}{\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu X^2}\frac{\cosh(Yn)}{\cosh(Yh)}\right.$$

$$+\frac{c_2}{\nu}\frac{k_{11}\cos t+k_{12}\sin t}{(y_1^2+z_1^2)(G^2+H^2)}+\frac{1}{\nu(x_1-x_2)}\left[\frac{e^{x_1t}c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}\right.$$

$$\left.-\frac{e^{x_2t}c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\frac{\cosh(\alpha n)}{\cosh(\alpha h)}+\frac{\pi}{h^2}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}(2r_2+1)}{\alpha_1^2}$$

$$\times\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]\left[\frac{e^{x_7t}(1+x_7\tau)^2}{[l+(1+x_7\tau)^2]}\frac{c_1(x_7^2+1)+c_2x_7^2}{x_7(x_7^2+1)}\right.$$

$$\left.\left.+\frac{e^{x_8t}(1+x_8\tau)^2}{[l+(1+x_8\tau)^2]}\frac{c_1(x_8^2+1)+c_2x_8^2}{x_8(x_8^2+1)}\right]\right\}$$

$$v_b(s,n,t)=\frac{c_1}{\nu X^2}\left[\frac{cosh(Xh)-cosh(Xs)}{cosh(Xh)}\right]+\frac{c_2}{\nu}\left[\frac{M_1\cos t+M_2\sin t}{(y_1^2+z_1^2)(C^2+D^2)(1+\tau^2)}\right]$$

$$+\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2h}s\right]\left[\frac{e^{x_3t}(1+x_3\tau)}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}\right.$$

$$\left.+\frac{e^{x_4t}(1+x_4\tau)}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]+\frac{2}{\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)$$

$$\times\left\{\frac{c_1}{\nu Y^2}\frac{\cosh(Yn)}{\cosh(Yh)}+\frac{c_2}{\nu}\frac{M_3\cos t+M_4\sin t}{(s_1^2+t_1^2)(G^2+H^2)(1+\tau^2)}\right.$$

$$+\frac{1}{\nu(x_5-x_6)}\left[\frac{e^{x_5t}c_1(x_5^2+1)+c_2x_5^2}{x_1(x_5^2+1)}-\frac{e^{x_6t}c_1(x_6^2+1)+c_2x_6^2}{x_2(x_6^2+1)}\right]$$

$$-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]\left[\frac{e^{x_7t}(1+x_7\tau)}{[l+(1+x_7\tau)^2]}\frac{c_1(x_7^2+1)+c_2x_7^2}{x_7(x_7^2+1)}\right.$$

$$\left.\left.+\frac{e^{x_8t}(1+x_8\tau)}{[l+(1+x_8\tau)^2]}\frac{c_1(x_8^2+1)+c_2x_8^2}{x_4(x_8^2+1)}\right]\right\}$$

$$+\frac{2\pi}{h^2}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu X^2Y^2}\frac{\cosh(Yn)}{\cosh(Xh)\cosh(Yh)}\right.$$

$$+\frac{c_2}{\nu}\frac{M_5\cos t+M_6\sin t}{(y_1^2+z_1^2)(s_1^2+t_1^2)(G^2+H^2)(C^2+D^2)(1+\tau^2)}$$

$$+\frac{1}{\nu(x_1-x_2)}\left[\frac{e^{x_1t}c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}-\frac{e^{x_2t}c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\frac{\cosh(\alpha n)}{\alpha^2\cosh(\alpha h)}$$

$$-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cosh(\beta n)}{\beta^2\cosh(\beta h)}\left[\frac{e^{x_3t}(1+x_3\tau)}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}\right.$$

$$\left.+\frac{e^{x_4t}(1+x_4\tau)}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]+\frac{(-1)^{r_1}}{\Gamma^2\nu(x_5-x_6)}\left[\frac{e^{x_5t}c_1(x_5^2+1)+c_2x_5^2}{x_5(x_5^2+1)}\right.$$

$$\left.-\frac{e^{x_6t}c_1(x_6^2+1)+c_2x_6^2}{x_6(x_6^2+1)}\right]-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]}{\alpha_2\cos(\alpha_1h)}$$

$$\left.\times\left[\frac{e^{x_7t}(1+x_7\tau)}{[l+(1+x_7\tau)^2]}\frac{c_1(x_7^2+1)+c_2x_7^2}{x_7(x_7^2+1)}+\frac{e^{x_8t}(1+x_8\tau)}{[l+(1+x_8\tau)^2]}\frac{c_1(x_8^2+1)+c_2x_8^2}{x_8(x_8^2+1)}\right]\right\}$$

$$-\frac{2}{\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu X^2}\frac{\cosh(Yn)}{\cosh(Yh)}+\frac{c_2}{\nu}\frac{M_7\cos t+M_8\sin t}{(y_1^2+z_1^2)(G^2+H^2)(1+\tau^2)}\right.$$

$$+\frac{1}{\nu(x_1-x_2)}\left[\frac{e^{x_1t}c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}-\frac{e^{x_2t}c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\frac{\cosh(\alpha n)}{\cosh(\alpha h)}$$

$$+\frac{\pi}{h^2}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}(2r_2+1)}{\alpha_2}\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]$$
$$\times\left[\frac{e^{x_7 t}(1+x_7\tau)}{[l+(1+x_7\tau)^2]}\frac{c_1(x_7^2+1)+c_2x_7^2}{x_7(x_7^2+1)}+\frac{e^{x_8 t}(1+x_8\tau)}{[l+(1+x_8\tau)^2]}\frac{c_1(x_8^2+1)+c_2x_8^2}{x_8(x_8^2+1)}\right]\Bigg\}$$

Shearing Stress (Skin Friction). The Shear stress at the boundaries $s = h,\ \ s=-h$ and $n=h,\ \ n=-h$ are given by

$$D_{h,n}=\frac{c_1\mu}{\nu X}\frac{\sinh(Xh)}{\cosh(Xh)}-\frac{c_2\mu}{\nu(y_1^2+z_1^2)(C^2+D^2)}[\cos t(Q_1P_1+Q_2P_2)+\sin t(Q_1P_2$$
$$-Q_2P_2)]+\frac{2\mu}{h}\left[\frac{e^{x_1 t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}+\frac{e^{x_2 t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\right.$$
$$\left.\times\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]-\frac{2\mu}{h}\left\{\frac{c_1}{\nu Y^2}\frac{\cosh(Yn)}{\cosh(Yh)}+\frac{c_2}{\nu}\frac{k_3\cos t+k_4\sin t}{(s_1^2+t_1^2)(G^2+H^2)}\right.$$
$$-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]\left[\frac{e^{x_3 t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\right.$$
$$\left.\left.\times\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}+\frac{e^{x_4 t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\}$$
$$-\frac{2\pi^2\mu}{h^3}\sum_{r_1=0}^{\infty}(-1)^{r_1}{r_1}^2\left\{\frac{c_1}{\nu X^2Y^2}\frac{\cosh(Yn)}{\cosh(Xh)\cosh(Yh)}+\frac{c_2}{\nu}\right.$$
$$\times\frac{L_5\cos t+L_6\sin t}{(y_1^2+z_1^2)(s_1^2+t_1^2)(G^2+H^2)(C^2+D^2)}-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cosh(\beta n)}{\beta^2\cosh(\beta h)}$$
$$\times\left[\frac{e^{x_1 t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}+\frac{e^{x_2 t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]$$
$$-\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]}{\alpha_2\cos(\alpha_1 r)}\left[\frac{e^{x_3 t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}\right.$$
$$\left.\left.+\frac{e^{x_4 t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\}-\frac{2\mu}{h}\sum_{r_1=0}^{\infty}(-1)^{r_1}\left\{\frac{c_1}{\nu X^2}\frac{\cosh(Yn)}{\cosh(Yh)}\right.$$
$$+\frac{c_2}{\nu}\frac{k_{11}\cos t+k_{12}\sin t}{(y_1^2+z_1^2)(G^2+H^2)}+\frac{\pi}{h^2}\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]$$
$$\left.\times\left[\frac{e^{x_3 t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}+\frac{e^{x_4 t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\}$$

$$
\begin{aligned}
D_{-h,n} &= -\frac{c_1\mu}{\nu X}\frac{\sinh(Xh)}{\cosh(Xh)} - \frac{c_2\mu}{\nu(y_1^2+z_1^2)(C^2+D^2)}\left[\cos t(-Q_1P_1 - Q_2P_2)\right. \\
&+ \left.\sin t(-Q_1P_2 + Q_2P_1)\right] - \frac{2\mu}{h}\left[\frac{e^{x_1 t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}\right. \\
&+ \left.\frac{e^{x_2 t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right] - \frac{2\mu}{h}\left\{\frac{c_1}{\nu Y^2}\frac{\cosh(Yn)}{\cosh(Yh)}\right. \\
&+ \frac{c_2}{\nu}\frac{k_3\cos t + k_4\sin t}{(s_1^2+t_1^2)(G^2+H^2)} - \frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\cos\left[\frac{(2r_2+1)\pi}{2h}n\right] \\
&\times\left[\frac{e^{x_3 t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)} + \frac{e^{x_4 t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\times\right. \\
&\left.\left.\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\} - \frac{2\pi^2\mu}{h^3}\sum_{r_1=0}^{\infty}(-1)^{r_1}{r_1}^2\left\{\frac{c_1}{\nu X^2Y^2}\frac{\cosh(Yn)}{\cosh(Xh)\cosh(Yh)}\right. \\
&+ \frac{c_2}{\nu}\frac{L_5\cos t + L_6\sin t}{(y_1^2+z_1^2)(s_1^2+t_1^2)(G^2+H^2)(C^2+D^2)} - \frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)} \\
&\times\frac{\cosh(\beta n)}{\beta^2\cosh(\beta h)}\left[\frac{e^{x_1 t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)} + \frac{e^{x_2 t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\right. \\
&\times\left.\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right] - \frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\cos\left[\frac{(2r_2+1)\pi}{2r}n\right]}{\alpha_2\cos(\alpha_1 r)}\left[\frac{e^{x_3 t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\right. \\
&\times\left.\left.\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)} + \frac{e^{x_4 t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\} \\
&+ \frac{2\mu}{h}\sum_{r_1=0}^{\infty}(-1)^{r_1}\left\{\frac{c_1}{\nu X^2}\frac{\cosh(Yn)}{\cosh(Yh)} + \frac{c_2}{\nu}\frac{k_{11}\cos t + k_{12}\sin t}{(y_1^2+z_1^2)(G^2+H^2)}\right. \\
&+ \frac{\pi}{h^2}\sum_{r_2=0}^{\infty}(-1)^{r_2}(2r_2+1)\cos\left[\frac{(2r_2+1)\pi}{2h}n\right]\left[\frac{e^{x_3 t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\right. \\
&\times\left.\left.\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)} + \frac{e^{x_4 t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
D_{s,h} &= \frac{2\mu}{\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu Y}\frac{\sinh(Yh)}{\cosh(Yh)} + \frac{c_2}{\nu(s_1^2+t_1^2)(G^2+H^2)}\right. \\
&\times\left[\cos t(q_1O_1 + q_2O_2) + \sin t(q_1O_2 - q_2O_1)\right] \\
&+ \frac{2}{h}\left[\frac{e^{x_3 t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)} + \frac{e^{x_4 t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\right.
\end{aligned}
$$

$$
\times \left.\left. \frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\} + \frac{2\pi\mu}{h^2}\sum_{r_1=0}^{\infty} r_1 \sin\left(\frac{r_1\pi}{h}s\right)
$$

$$
\times \left\{ \frac{c_1}{\nu X^2 Y}\frac{\sinh(Yh)}{\cosh(Xh)\cosh(Yh)} + \frac{c_2}{\nu(y_1^2+z_1^2)(s_1^2+t_1^2)(C^2+D^2)(G^2+H^2)} \right.
$$

$$
\times[\cos t(c_{11}O_1+d_{11}O_2)+\sin t(c_{11}O_2+d_{11}O_1)] - \frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\sinh(\beta h)}{\beta\cosh(\beta h)}
$$

$$
\times \left[\frac{e^{x_1t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)} + \frac{e^{x_2t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]
$$

$$
+\frac{2}{h\alpha_2\cos(\alpha_1 h)}\left[\frac{e^{x_3t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)} + \frac{e^{x_4t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\right.
$$

$$
\times \left.\left.\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\} - \frac{2\mu}{\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1 Y}{\nu X^2}\frac{\sinh(Yh)}{\cosh(Yh)}\right.
$$

$$
+\frac{c_2}{\nu(y_1^2+z_1^2)(G^2+H^2)}[\cos t(u_1O_1+u_2O_2)+\sin t(u_1O_2-u_2O_1)]
$$

$$
-\frac{\pi^2}{2h^3}\sum_{r_2=0}^{\infty}(2r_2+1)^2\left[\frac{e^{x_3t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)} + \frac{e^{x_4t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\right.
$$

$$
\times \left.\left.\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\} + \frac{c_1\mu}{\nu X}\frac{\sinh(Xs)}{\cosh(Xh)} - \frac{c_2\mu}{\nu(y_1^2+z_1^2)(C^2+D^2)}[\cos t(Q_1v_1
$$

$$
+Q_2v_2)+\sin t(Q_1v_2-Q_2v_1)] + \frac{2\mu}{h}\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2h}s\right]
$$

$$
\times \left[\frac{e^{x_1t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)} + \frac{e^{x_2t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]
$$

$$
-\frac{2\mu}{h}\sum_{r_1=0}^{\infty}(-1)^{r_1}\cos\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu Y^2} + \frac{c_2}{\nu(s_1^2+t_1^2)(G^2+H^2)}[\cos t(q_1O_1\right.
$$

$$
\left.+q_2O_2)+\sin t(q_1O_2-q_2O_1)]\right\} - \frac{2\pi^2\mu}{h^3}\sum_{r_1=0}^{\infty}{r_1}^2\cos\left(\frac{r_1\pi}{h}s\right)
$$

$$
\times\left\{\frac{c_1}{\nu X^2Y^2\cosh(Xr)} + \frac{c_2}{\nu}\frac{\cos t(c_{11}O_1+d_{11}O_2)+\sin t(c_{11}O_2-d_{11}O_1)}{(y_1^2+z_1^2)(s_1^2+t_1^2)(C^2+D^2)(G^2+H^2)}\right.
$$

$$
+\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)\beta^2}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}\right.
$$

$$
\left.\left.+\frac{e^{x_2t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\right\} + \frac{2\mu}{h}\sum_{r_1=0}^{\infty}\cos\left(\frac{r_1\pi}{h}s\right)
$$

$$\times\left\{\frac{c_1}{\nu X^2}+\frac{c_2}{\nu}\frac{\cos t(u_1O_1+u_2O_2)+\sin t(u_1O_2-u_2O_1)}{(y_1^2+z_1^2)(G^2+H^2)}\right\}$$

$$\begin{aligned}
D_{s,-h} &= \frac{2\mu}{\pi}\sum_{r_1=0}^{\infty}\frac{(-1)^{r_1}}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{-\frac{c_1}{\nu Y}\frac{\sinh(Yh)}{\cosh(Yh)}+\frac{c_2}{\nu(s_1^2+t_1^2)(G^2+H^2)}\right.\\
&\times[-\cos t(q_1O_1+q_2O_2)-\sin t(q_1O_2-q_2O_1)]-\frac{2}{h}\left[\frac{e^{x_3t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\right.\\
&\times\left.\left.\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}+\frac{e^{x_4t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\}\\
&+\frac{2\pi\mu}{h^2}\sum_{r_1=0}^{\infty}r_1\sin\left(\frac{r_1\pi}{h}s\right)\left\{-\frac{c_1}{\nu X^2Y}\frac{\sinh(Yh)}{\cosh(Xh)\cosh(Yh)}\right.\\
&+\frac{c_2}{\nu(y_1^2+z_1^2)(s_1^2+t_1^2)(C^2+D^2)(G^2+H^2)}[-\cos t(c_{11}O_1+d_{11}O_2)\\
&-\sin t(c_{11}O_2-d_{11}O_1)]+\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)}\frac{\sinh(\beta h)}{\beta\cosh(\beta h)}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\right.\\
&\times\left.\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}+\frac{e^{x_2t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\\
&-\frac{2}{h\alpha_2\cos(\alpha_1h)}\left[\frac{e^{x_3t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}+\frac{e^{x_4t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\right.\\
&\times\left.\left.\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\}-\frac{2\mu}{\pi}\sum_{r_1=0}^{\infty}\frac{1}{r_1}\sin\left(\frac{r_1\pi}{h}s\right)\left\{-\frac{c_1Y}{\nu X^2}\frac{\sinh(Yh)}{\cosh(Yh)}\right.\\
&+\frac{c_2}{\nu(y_1^2+z_1^2)(G^2+H^2)}[-\cos t(u_1O_1+u_2O_2)-\sin t(u_1O_2-u_2O_1)]\\
&+\frac{\pi^2}{2h^3}\sum_{r_2=0}^{\infty}(2r_2+1)^2\left[\frac{e^{x_3t}(1+x_3\tau)^2}{[l+(1+x_3\tau)^2]}\frac{c_1(x_3^2+1)+c_2x_3^2}{x_3(x_3^2+1)}+\frac{e^{x_4t}(1+x_4\tau)^2}{[l+(1+x_4\tau)^2]}\right.\\
&\times\left.\left.\frac{c_1(x_4^2+1)+c_2x_4^2}{x_4(x_4^2+1)}\right]\right\}+\frac{c_1\mu}{\nu X}\frac{\sinh(Xs)}{\cosh(Xh)}-\frac{c_2\mu}{\nu(y_1^2+z_1^2)(C^2+D^2)}[\cos t(Q_1v_1\\
&+Q_2v_2)+\sin t(Q_1v_2-Q_2v_1)]+\frac{2\mu}{h}\sum_{r_2=0}^{\infty}(-1)^{r_2}\sin\left[\frac{(2r_2+1)\pi}{2h}s\right]\\
&\times\left[\frac{e^{x_1t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}+\frac{e^{x_2t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\\
&-\frac{2\mu}{h}\sum_{r_1=0}^{\infty}(-1)^{r_1}\cos\left(\frac{r_1\pi}{h}s\right)\left\{\frac{c_1}{\nu Y^2}+\frac{c_2}{\nu(s_1^2+t_1^2)(G^2+H^2)}[-\cos t(q_1O_1\right.
\end{aligned}$$

$$+ q_2O_2) - \sin t(q_1O_2 - q_2O_1)]\} - \frac{2\pi^2\mu}{h^3}\sum_{r_1=0}^{\infty} {r_1}^2 \cos\left(\frac{r_1\pi}{h}s\right)$$
$$\times \left\{\frac{c_1}{\nu X^2Y^2\cosh(Xh)} + \frac{c_2}{\nu}\frac{-\cos t(c_{11}O_1 + d_{11}O_2) - \sin t(c_{11}O_2 - d_{11}O_1)}{(y_1^2+z_1^2)(s_1^2+t_1^2)(C^2+D^2)(G^2+H^2)}\right.$$
$$+\frac{4}{\pi}\sum_{r_2=0}^{\infty}\frac{(-1)^{r_2}}{(2r_2+1)\beta^2}\left[\frac{e^{x_1t}(1+x_1\tau)^2}{[l+(1+x_1\tau)^2]}\frac{c_1(x_1^2+1)+c_2x_1^2}{x_1(x_1^2+1)}\right.$$
$$\left.\left.+\frac{e^{x_2t}(1+x_2\tau)^2}{[l+(1+x_2\tau)^2]}\frac{c_1(x_2^2+1)+c_2x_2^2}{x_2(x_2^2+1)}\right]\right\} + \frac{2\mu}{h}\sum_{r_1=0}^{\infty}\cos\left(\frac{r_1\pi}{h}s\right)$$
$$\times\left\{\frac{c_1}{\nu X^2} + \frac{c_2}{\nu}\frac{[-\cos t(u_1O_1+u_2O_2) - \sin t(u_1O_2 - u_2O_1)]}{(y_1^2+z_1^2)(G^2+H^2)}\right\}$$

where

$A = \cosh(y_2h)\cos(z_2h) - \cosh(y_2s)\cos(z_2s)$,

$B = \sinh(y_2h)\sin(z_2h) - \sinh(y_2s)\sin(z_2s)$,

$C = \cosh(y_2h)\cos(z_2h)$, $D = \sinh(y_2h)\sin(z_2h)$, $E = \cosh(s_2n)\cos(t_2n)$,

$F = \sinh(s_2n)\sin(t_2n)$, $G = \cosh(s_2h)\cos(t_2h)$, $H = \sinh(s_2h)\sin(t_2h)$,

$L_1 = AC + BD$, $L_2 = BC - AD$, $L_3 = EG + FH$, $L_4 = FG - EH$,

$L_5 = k_9s_1 - k_{10}t_1$, $L_6 = k_{10}s_1 + k_9t_1$, $M_1 = k_1 - \tau k_2$, $M_2 = k_2 + \tau k_1$,

$M_3 = k_3 + \tau k_4$, $M_4 = k_4 - \tau k_3$, $M_5 = L_5 - \tau L_6$, $M_6 = L_6 + \tau L_5$,

$M_7 = k_{11} - \tau k_{12}$, $M_8 = k_{12} + \tau k_{11}$, $O_1 = s_2a_{11} - t_2b_{11}$, $O_2 = s_2b_{11} + t_2a_{11}$,

$P_1 = -y_2g_{11} + z_2h_{11}$, $P_2 = -y_2h_{11} - z_2g_{11}$, $Q_1 = Cy_1 + Dz_1$, $Q_2 = Dy_1 - Cz_1$,

$X = \sqrt{C_r}$, $Y = \sqrt{C_r + \frac{r_1^2\pi^2}{h^2}}$, $a_0 = \tau$, $a_1 = 4h^2\tau$, $a_2 = h^2\tau$, $a_3 = 4h^2\tau$

$b_0 = (C_r\nu\tau + 1 + l)$, $b_1 = (C_r\nu\tau + 1 + l)4h^2$,

$b_2 = (C_r\nu\tau + l + 1)4h^2 + (2r_2+1)^2\pi^2\nu\tau$,

$b_3 = (C_r\nu\tau + l + 1)4h^2 + 4r_1^2\pi^2\nu\tau + (2r_2+1)^2\pi^2\nu\tau$, $c_0 = C_r\nu$,

$c_1 = 4h^2C_r\nu + (2r_2+1)^2\pi^2\nu$, $c_2 = C_r\nu h^2 + r_1^2\pi^2\nu$, $c_3 = 4h^2C_r\nu + 4r_1^2\pi^2\nu$,

$k_1 = L_1y_1 - L_2z_1$, $k_2 = L_2y_1 + L_1z_1$, $k_3 = L_3s_1 - L_4t_1$, $k_4 = L_4s_1 + L_3t_1$,

$k_5 = CG - DH$, $k_6 = DG + CH$, $k_7 = Ek_5 + Fk_6$, $k_8 = Fk_5 - Ek_6$,

$k_9 = k_7y_1 - k_8z_1$, $k_{10} = k_8y_1 + k_7z_1$, $k_{11} = L_3y_1 - L_4z_1$, $k_{12} = L_4y_1 + L_3z_1$,

$$q_1 = Gs_1 + Ht_1, \quad q_2 = Hs_1 - Gt_1, \quad s_1 = \frac{C_r \nu h^2(1+\tau^2) + r_1^2\pi^2\nu(1+\tau^2) + lh^2\tau}{\nu h^2(1+\tau^2)},$$

$$s_2 = \sqrt{\frac{s_1 + \sqrt{s_1^2 + t_1^2}}{2}}, \quad t_1 = \frac{1 + l + \tau^2}{\nu(1+\tau^2)}, \quad t_2 = \sqrt{\frac{-s_1 + \sqrt{s_1^2 + t_1^2}}{2}},$$

$$u_1 = Gy_1 + Hz_1, \quad u_2 = Hy_1 - Gz_1, \quad v_1 = -y_2 e_{11} + z_2 f_{11}, \quad v_2 = -y_2 f_{11} - z_2 e_{11},$$

$$x_1 = \frac{-b_0 + \sqrt{b_0^2 - 4a_0c_0}}{2a_0}, \quad x_2 = \frac{-b_0 - \sqrt{b_0^2 - 4a_0c_0}}{2a_0},$$

$$x_3 = \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1}, \quad x_4 = \frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1},$$

$$x_5 = \frac{-b_2 + \sqrt{b_2^2 - 4a_2c_2}}{2a_2}, \quad x_6 = \frac{-b_2 - \sqrt{b_2^2 - 4a_2c_2}}{2a_2},$$

$$x_7 = \frac{-b_3 + \sqrt{b_3^2 - 4a_3c_3}}{2a_3}, \quad x_8 = \frac{-b_3 - \sqrt{b_3^2 - 4a_3c_3}}{2a_3}, \quad y_1 = \frac{C_r\nu(1+\tau^2) + l\tau}{\nu(1+\tau^2)},$$

$$y_2 = \sqrt{\frac{y_1 + \sqrt{y_1^2 + z_1^2}}{2}}, \quad z_1 = \frac{1 + l + \tau^2}{\nu(1+\tau^2)}, \quad z_2 = \sqrt{\frac{-y_1 + \sqrt{y_1^2 + z_1^2}}{2}},$$

$$a_{11} = \sinh(s_2h)\cos(t_2h), \quad b_{11} = \sin(t_2h)\cosh(s_2h), \quad h_{11} = \cosh(y_2h)\sin(z_2h),$$

$$c_{11} = (k_5y_1 + k_6z_1)s_1 + (k_6y_1 - k_5z_1)t_1, \quad d_{11} = (k_6y_1 - k_5z_1)s_1 - (k_5y_1 + k_6z_1)t_1,$$

$$e_{11} = \sinh(y_2s)\cos(z_2s), \quad f_{11} = \cosh(y_2s)\sin(z_2s), \quad g_{11} = \sinh(y_2h)\cos(z_2h),$$

$$\beta = \sqrt{\frac{4r_1^2\pi^2 - (2r_2+1)^2\pi^2}{4h^2}}, \quad \alpha = \sqrt{\frac{r_1^2\pi^2}{h^2}}, \quad \alpha_1 = \sqrt{\frac{4r_1^2\pi^2 + (2r_2+1)^2\pi^2}{4h^2}},$$

$$\alpha_2 = -\left[\frac{4r_1^2\pi^2 + (2r_2+1)^2\pi^2}{4h^2}\right], \quad \Gamma = \tfrac{ir_1\pi}{h}.$$

Conclusion

The figures 3 and 4 represents the velocity profiles for both fluid and dust particles respectively, which are paraboloid in nature. Also the velocity of both fluid and dust particles, move with the greater velocity nearer to the axis of flow. Further it shows that if the dust is very fine i.e., mass of the dust particles is negligibly small then the relaxation time of dust particle decreases and ultimately as $\tau \to 0$ the velocities of fluid and dust particles will be the same. It can also be noticed from the figures 3 and 4 that the velocities of both fluids and dust going to be decrease as time t increases.

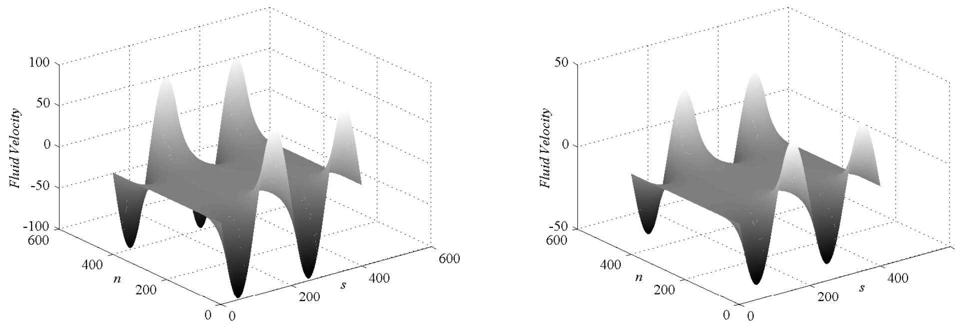

Figure 3. Variation of fluid velocity with s and n (for $t = 1.0$ & $t = 3.0$)

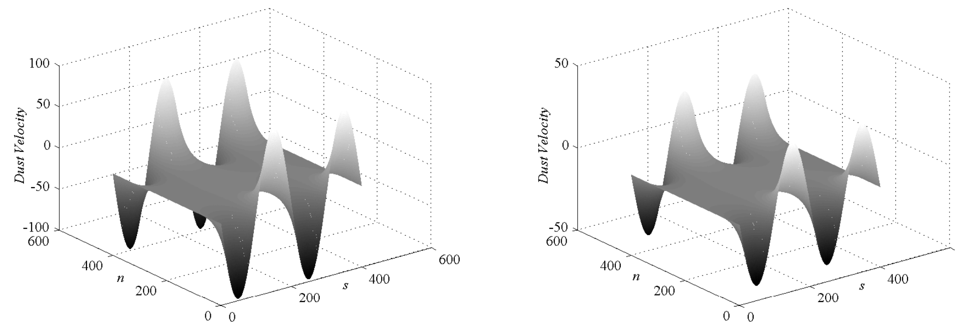

Figure 4. Variation of dust velocity with s and n (for $t = 1.0$ & $t = 3.0$)

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