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Metamathematics in Reporting of Research

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Abstract
Corresponding to the three meanings of the prepositional prefix 'meta', three types of metamathematics (MM) are conceived. They are captioned as MM-I, MM-II, MM-III. The topics for illustration of MM are chosen from Relativity, as well as other primary subjects. It is argued that (i) MM-I does not exist, when 'meta' stands for 'beyond'. (ii) MM-II is unreadable when 'meta' indicates 'behind'. (iii) MM-III is essential and important for a lively reporting of research and for thrilling teaching, when 'meta' means 'about'. In this paper we elaborate MM-III through the four types of mathematical/ research patterns- after substituting ideas for calculations. Examples are cited to demonstrate (1) Significant research differentiated from Ignorable research (2) Exciting mathematics distinguished from aesthetic mathematics. The appreciation of mathematics (MM-III) contains humor and thrills, which are absent in mathematics per se (definition, axiom, theorem, proof).

On the eve of National Year of Mathematics (2012) in India, an appeal is made here to develop consciously the culture of metamathematics, for making research attractive and improve the Science Citation Index in Indian publications and for delivering inspiring professional talks.

1. Introduction

Metamathematics in a research paper : Every research paper has two parts: (i) Formal part consisting of definitions, alphabetical /non-alphabetical symbols, technical words (without synonyms and antonyms), theorem, proof, corollary. It deals with successful calculations/ deductions all in technical jargon. Logicians refer to it as *Schemata*. (ii) Informal part consisting of "Introduction and Conclusion". This part is metamathematics (appreciation of mathematics reported in the paper), which reflects the motivation, the spirit, and the flavor of formal part. The informal part is

subjective, but attracts the connoisseurs of research: the reviewers, critics, and fellow mathematicians. In journalistic jargon, metamathematics can be referred as 'marketing mathematics'. The use of English words is the essential part of this exposition, while the technical jargon is just incidental. Without the informal part, we cannot follow the formal part; that is, "*there is no mathematics without metamathematics*" [Lieber and Lieber, 1967]. For a new research scholar, the formal part dominates the informal part of research; while for a professional researcher the informal dominates the formal. Thus metamathematics reflects the mathematical experience of the researcher.

History and philosophy of the topic of research (the 'Ph' factor in 'Ph.D.' degree), strategy for solution, type of abstraction, type of generalization used in the paper are usually included in 'Introduction'. An appreciation of the successful calculations (vide the four types of mathematical patterns), plausible future applications, possible modifications of the premise and the *ism* (perspective) of the paper are included in 'Conclusion'. For a new research scholar the drafting of Introduction and conclusion are difficult. The research guide comes to his rescue. However a concerted effort in the direction of metamathematics will help in the increase of Science Citation Index of the paper.

2. Metamathematics: An Exposition

The three meanings of the prefix 'meta' are the prepositions: 'beyond', 'behind' and 'about'. Accordingly MM-I is devoted to the question of whether there is any branch of knowledge, which is beyond mathematics. Logical symbolism is the essence of MM-II, since 'logic' is behind mathematics, as the foundation. MM-III is 'about' mathematics (or 'after' calculations) and it plays a significant role in presentation of mathematical teaching and research. A new researcher hides MM in his M (calculations/ mathematics); that is, M dominates MM. For an established researcher the converse happens: MM dominates M. Thus MM is a tribute to the mathematical experience of the senior researcher.

An Exposition of Metamathematics - I

We consider the analogous topics metaphysics and metaengineering with examples, where 'meta' refers to 'beyond'. We start with the etymology of the word 'engineering' as 'profiteering using ingenuity'. From the sentence in engineering:

$$\text{"}A \text{ rocket moves at the rate of } 6000 \text{ miles per hour"}, \qquad (A)$$

if we drop the first seven words, the resulting phrase is

$$\text{"6000 } \textit{miles per hour}\text{"}. \qquad (B)$$

This is a common expression in physics. Consequently we observe that

$$\textit{metaengineering is physics.} \qquad (I)$$

This supports the view that 'engineering is the realization of physics'.

From the phrase (B), if we drop the three words 'miles per hour', we are left with the number

$$\text{"6000"}. \qquad (C)$$

We generalize this situation by replacing 6000 by x and observe that mathematics is beyond physics or

$$\textit{metaphysics is mathematics.} \qquad (II)$$

From the sentences (I), (II) we infer that mathematics is meta-metaengineering! If we delete "6000", then there exists nothing in (C). We then claim that metamathematics-I does not exist! In other words, mathematics is the ultimate subject in abstraction; it explains why United Nations Educational Scientific and Cultural Organization, chose the caption WORLD MATHEMATICS YEAR 2000, to celebrate the new millennium (2001-3000 A.D.)!

An Exposition of Metamathematics- II

Consider the logical statements in analysis (complex/ real):

(i) $\forall z\ \exists p\ \exists u$
$[(p = |p|) \wedge (|u| = 1) \wedge (z = pu)]$

(ii) $\forall x\ \exists y\ \ni\ x \geq 0 \implies y^2 = x.$

Since no one thinks in terms of

$$\forall, \quad \exists, \quad \rightarrow, \quad \leftrightarrow, \quad \nexists, \quad \epsilon, \quad \therefore,$$

the logical symbolism becomes a cumbersome code and creates unpleasantness to the mind through thought blockades. Replacement of the symbols of logic by words, promotes communication of ideas; accordingly we describe (i) and (ii) in words as follows.

(i') "Every complex number is the product of a non-negative real number and a complex number of modulus one". (ii') "Every non-negative real number has a square

root". It is interesting to note, that the book Principia Mathematica by Russell and Whitehead is considered as an

"outstanding example of unreadable masterpiece"

because the text is filled with logical symbols and no words! After 362 pages of abstract notation and no words, they prove $1 + 1 = 2$! Thus MM-II runs the risk of incomprehensibility and avoidable jargon.

3. An Exposition of Metamathematics - III

We develop the theme 'about mathematics'. How to appreciate and enjoy mathematical results 'after completing the calculations' or 'after proving a theorem' in a class room/ research paper. The delineation of the four Patterns of Mathematics Discourse/ Research Results follows.

Mathematical Pattern 1. SIMPLICITY OUT OF SIMPLICITY

In the context of mathematics, 'Simplicity' means 'predictability', 'arrangement'; and 'Complexity' means 'confusion', 'mixedupness', and 'randomness' (apparent).

(i) Aesthetic mathematics/ research

Calculations: From the equation of a sphere

$$x^2 + y^2 + z^2 = 1 \tag{1}$$

we can deduce the equation of the tangent plane to the sphere at the point (x_1, y_1, z_1) as

$$xx_1 + yy_1 + zz_1 = 1 \tag{2}$$

Substituting ideas for calculations: Equation (1) is symmetric and homogeneous on the left hand side and the right hand side has a simple number. Equation (2) has similar nice features. We have simple Eq. (2) out of another simple Eq. (1). We call this derivation as the idea "simplicity out of simplicity". It is pleasing to the eye and the mind; so we call the derivation of (2) from (1) as 'aesthetic mathematics'. This characterization of (2) from (1) is an example of MM III. When Descarte first derived (2) from (1) in 16th century, it was rightly termed "aesthetic research".

Algebra abounds with such aesthetic circumstances. The axioms of a group structure are an instance of simplicity (associativity, existence of the unit element, existence of the inverse). If we combine them with another simplicity (*the axioms of a field*)—in a certain way — we get the axioms of yet another simplicity — the famous algebraic structure vector space, which is an "ubiquitous discipline appearing and

reappearing in science and pseudo science", as eulogized by Herstein. We present this nice situation as aesthetic mathematics:

$$Group\ (Field)\ :\ Vector\ Space.$$

Thus 'simplicity out of (simplicity out of simplicity)' opened the flood gates of research in algebra:

$$Group\ (Ring)\ :\ Module,$$

$$Ring\ (Ring)\ :\ Algebra$$

$$Topological\ Vector\ Space\ (Topological\ Vector\ Space)\ :\ Fiber\ Bundle.$$

As of now there are 1131 algebras [Grinstein and Lipsey 2001], [Radhakrishna 2008]. This methodology of 'simplicity out of simplicity' offers a clue for generating new research results out of the old! Such a discovery of aesthetic mathematics catapults the researcher into an instant celebrity.

Mathematical Pattern 2: COMPLEXITY OUT OF SIMPLICITY

(ii) Exciting mathematics/ research

We consider the following results:

$$\sqrt{2} = 1.41421356237...$$

$$e = 2.7182818828...$$

$$\pi = 3.141592653....$$

Substitution of ideas for calculations (metamathematics): On the left hand side of the three equalities there is simplicity. However on the right hand sides there is apparent complexity. There is no predictability even after calculating a thousand decimal places! Such results are identified as "complexity out of simplicity". They are rare in history of mathematics and also exciting. We praise the results as exciting mathematics. When Pythagoras found the first result, he celebrated the exciting event, by sacrificing a hecatomb of oxen!

Mathematical pattern 3: COMPLEXITY OUT OF COMPLEXITY

(iii) Ignorable research/ mathematics

Guinness book of world records in mathematics (1980) mentions the following instances:

[1] Shakuntala Devi and computer were given to multiply two 14 digit numbers and Devi gave the product correctly well before the computer could announce the answer (Imperial College of Science and Technology, London):

$$7686369774870x2465099745779 = 1894766817799542646277 3730.$$

Here the left hand side and the right hand side both display complexity. No one will try to remember the result! It is just ignored as it does not improve our knowledge.
[2] In 1984 Rajan Mahadevan recited π to 31,940 decimal places on All India Radio. It is an example of tremendous memory power of the brain. This feat is *ignored* since it has no utility beyond 5 digits.
Substituting ideas for calculations: An arbitrary polynomial multiplied by another arbitrary polynomial produces a huge polynomial. This is ignorable work. Another example is 'dry water research'. One must be beware of such ignorable results and stop the temptation of generalizing them!

Mathematical pattern 4: SIMPLICITY OUT OF COMPLEXITY

(iv) Significant mathematics/ research

All theorems that are taught in the class rooms belong to this variety. We treasure them and repeatedly teach them for generations. We mention a famous theorem and how it brings in simplicity out of obviously complex situation.
Pythogoras theorem (4 century B.C.): In ANY triangle ABC, we have

$$c^2 = a^2 + b^2 - 2ab\ \cos C. \qquad (P)$$

The word 'any' suggests the complexity-the uncountable infinity of choices of the constants a, b, c. The simplicity is represented by the one formula (P). The teachers to be effective should be able to describe the chaos before the discovery/ invention of the theorem and the one pleasant simplicity achieved by the author, after completing the proof of the theorem. This famous theorem has now 370 proofs when $C = 90°$.

4. Beauty, Humor and Thrills in Metamathematics III

We cite some mathematical formulae and the corresponding metamathematical descriptions in a tabular form:

	MATHEMATICS (Mathematicians)	METAMATHEMATICS (Metamathematicians)
(1)	$e^{i\pi} + 1 = 0$ (EULER)	The Most Exciting Equation (BENJAMIN PIERCE)
(2)	$E = mc^2$ (EINSTEIN)	The Most Famous Equation (DAVID BODANIS)
(3)	$R^i_j - \frac{1}{2} R\, g^i_j = -8\pi G c^{-4} R T^i_j$ (EINSTEIN)	The Taj Mahal of Science (GEORGE GAMOW)
(4)	$R^i_j - \frac{1}{2} R\, g^i_j + \Lambda g^i_j = -8\pi G c^{-4} R T^i_j$ (EINSTEIN)	Himalayan Blunder (EINSTEIN)

Beauty in Metamathematics, not in Mathematics: A popular quotation reads "Beauty is not in the holder, but in the beholder". Beauty is not in the formula itself, but lies in metamathematics —the comments of the beholding mathematician. It is the metamathematics that inspires a student to take all pains to understand the formula and feel its greatness personally. Einstein has revised our universe with just his field equations 'the Taj Mahal of Science', inaugurating the mathematics of light, the tool of *self orthogonal vector fields*, and the mathematical study of Microcosmos and Macrocosmos, where high speeds comparable to the velocity of light exist. It is no wonder that the mathematician Einstein has been elected as the person of the second millennium (1001-2000 A.D.) by TIME magazine [Isaacson 2000].

Humor in Metamathematics, not in Mathematics: While logic, ingenuity, consistency, symbolism, technical terms, axioms, theorems, proofs and correct grammar

are essential features of mathematics prose, humor is not an integral part of mathematics. In fact humor belongs to metamathematics, when the prepositional prefix 'meta' has the meaning 'about' (but not 'behind' or 'beyond'). See Einstein's Himalayan blunder in the second column of the table cited above.

Thrills in Metamathematics: Excitement and pleasure are not in formulae or derivations as such; just as beauty is not in the holder. Beauty is in the beholder, the practitioner of mathematics. Often the source for thrills is from the historians/ philosophers of the subject. The four philosophies (each with its own definition of mathematics), the two dogmas of mathematics, the prediction of physical phenomenon (via thought experiments), provide items for the thrills (Radhakrishna, 2008).

5. Conclusion

Relevance of Metamathematics: The appreciation of mathematics by a great author/ teacher, which constitutes metamathematics, provides a strong motivation for students to understand and master the difficult technicalities of an argument. Sometimes witty observations of one author on the life and work of other researchers/authors (due to professional jealousy) produce humor like Leibnitz's comment on the co-invention of calculus by Newton: NEWTON WENT ON NOT NEW. To pep up listeners' attention in a class or a conference, the items of humor and thrills-which are integral part of metamathematics— (See Radhakrishna 2012), provide a veritable source! From Section 4, it is obvious that they are part of metamathematics, but not of mathematics. It is the art of lively teaching/oration. On the eve of National Year of Mathematics (2012) in India, an appeal is made here to develop consciously the *culture of metamathematics*, for enhancing the relevance/ spirit of research and improve the Science Citation Index in Indian publications.

Acknowledgment

It is a privilege to dedicate this paper to the memory of Prof. S. I. Hussain (1935-1994), who was one of the few Metamathematicians of venerable variety! His penchant for appreciation of mathematics was distinct not only in his popular lectures, but also in technical lectures in national/ international conferences. His research career started with Delhi university and sharpened in France, a famous place for metamathematics. I invited Prof. Hussain in 1976 to Shivaji university for a talk. He was influenced by the French culture so much that he chose to speak on 'Polycephalic mathematician' (many headed mathematician), who was born in 1939 (but who will not die), who writes for ever "*Elements of Mathematics*"— the most authoritative

survey of modern mathematics—, whose new terminology in Set theory is adopted by fellow mathematicians of the world, whose name is Greek, whose nationality is French, whose name is mentioned in inverted commas in all publications, and who does not exist!

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