J. T. S. ISSN : 0974-5428
Vol. 13 (2019), pp.81-90

FRW Models with Late-time Acceleration in presence of Interacting Dark Energy

S. K. J. Pacif[1] and L. K. Paikroy[2]

[1]Department of Mathematics, School of Advanced Sciences,
Vellore Institute of Technology, Vellore 632014, Tamil Nadu, India
[2]Department of Mathematics, Dalmia College, Raj Gangpur
Odisha 770017, India
e-mail: shibesh.math@gmail.com, lplpaikray@gmail.com
(Received: September 25, 2019, Accepted: December 1, 2019)

Abstract

In this paper, some FRW models exhibiting late-time acceleration are obtained in the presence of an interacting dark energy represented by a time varying cosmological constant in general theory of relativity. In order to solve the Einstein's field equations, we have considered a simple parametrization of the energy density ρ connecting radiation and matter dominated eras. Some features of the obtained models are then discussed.

Keywords: Dark Energy, cosmological constant, general relativity, accelerating universe.
2000 AMS Subject Classification: 83C15, 83F05.
PACS: 04.20.-q, 98.80.-k

1. Introduction

Cosmic acceleration is the most discussed area of research in the contemporary cosmological studies. The observation of high redshift supernovae of type Ia [1, 2] in the late 90's changed the view of the then cosmology of the expanding universe with deceleration due to attractive gravity. Since then, the idea of late-time cosmic acceleration has become an important topic in the scientific community with more robust analyses. But, the cause of acceleration is still a mystery and we are mostly in dark about the nature of this weird form of energy. However, it is assumed to be homogeneous with high negative pressure with increasing density dominating the energy budget in the Universe. This mysterious form of energy is generally termed as *dark energy* [3]. The candidature of dark energy is a debatable topic till. Various candidates of dark energy have been considered in the past two decades and a plethora of different dark energy models have been proposed [4, 5].

The simplest and most favoured candidate of dark energy is the Einstein's cosmological constant Λ and the ΛCDM model is also in good agreement with some present observations. However, it's non-dynamical nature of Λ, alternatives have been discussed such as scalar field models [6, 7, 8, 9, 10, 11]. However, the consideration of time-dependent cosmological constant has a long history before the discovery of cosmic acceleration in the context of solving the cosmological constant problem. In this paper, we have discussed a dark energy model by considering a time-dependent cosmological constant as a candidate of dark energy. Exact solution of Einstein's field equations are obtained by considering an ansatz describing the variation of energy density of matter connecting radiation and matter eras. The parametrization of cosmological parameters or the model independent way is the simple method to study any dark energy model (see [12] for various parametrization schemes).

The paper is organized as follows. The first section is introductory and discusses the present cosmological scenario. In the second section, we have discussed the Einstein's field equations in general relativity in presence of interacting dark energy. The third section discusses the solution and interpretation. Finally, we have concluded our results in the fourth section.

2. Field Equations

Let us considers the Robertson-Walker metric in the form,

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right], \tag{1}$$

which represents the homogeneous and isotropic Universe (at sufficiently large scales) in which the distribution of matter is represented by the energy-momentum tensor of a perfect fluid

$$T_{ij}^M = (\rho + p)\, U_i U_j + p\ g_{ij}\ , \tag{2}$$

where ρ is the energy density of the cosmic matter and p is its pressure, then the Einstein field equations

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} = -8\pi G T_{ij}^M\ , \tag{3}$$

yield the following two independent equations

$$8\pi G\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \tag{4}$$

$$8\pi G p = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}. \tag{5}$$

It is then easy to see that for normal matter with $p > 0$, $\rho > 0$, the equations (4) and (5) require $\ddot{R} < 0$ forever, regardless of the geometry of the universe. Though, very less is known about the dark energy, but it can be represented by a large-scale scalar field ϕ dominated either by potential energy or nearly constant potential energy. This kind of matter can have its energy-stress tensor in the form $T_{ij}^{DE} = (\rho_\phi + p_\phi)U_iU_j + p_\phi\ g_{ij}$ and with equation of state in the form $p_\phi = w_\phi \rho_\phi$, where w_ϕ is a function of time in general. Obviously, this describes a number of candidates for dark energy depending upon the dynamics of the field ϕ and its potential energy. For potential dominated scalar field, this reduces to the Einstein's cosmological constant Λ for which w_ϕ becomes -1. As we have discussed the simplest candidate of dark energy is the Einstein's cosmological constant Λ, hereafter, we introduce Λ into the field equations as an extra source. Moreover, dark energy can be introduced in Einstein's theory by replacing T_{ij}^M by T_{ij}^{total} in equation (3), where $T_{ij}^{total} = T_{ij}^M + T_{ij}^{DE} = (\rho_t + p_t)U_iU_j + p_t\ g_{ij}$ with the understanding that $\rho_t = \rho + \rho_\phi$ and $p_t = p + p_\phi$. In this case, equations (4) and (5) are modified as

$$8\pi G\rho_t = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \tag{6}$$

$$8\pi G p_t = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}. \tag{7}$$

Now, the Bianchi identities requires that T_{ij}^{total} must have a vanishing divergence. Without considering any additional assumption of minimal coupling between matter and dark energy believing the interaction is natural and is a fundamental principle [13] leading to,

$$\frac{d}{dt}\left(\rho_t R^3\right) + p_t \frac{dR^3}{dt} = 0. \tag{8}$$

3. Solution and Interpretation

We can see, there are only two independent equations (6) and (7) connecting four unknowns R, ρ, p and Λ. As we know the cosmological constant can also be represented as intrinsic energy density of vacuum as $\Lambda = 8\pi G\rho_v$, we consider ρ_v

in the following. For complete determination, two more equations are required, one of which we choose here as the usual barotropic equation of state

$$p = w\rho,\ \ w = \text{constant}. \tag{9}$$

Now the conservation equation (8) with $\rho = \rho_m + \rho_r$, ρ_m and ρ_r being the matter (rest mass) and radiation energy densities and ρ_v being the vacuum energy density, may be written as

$$\dot{\rho}_v + \frac{1}{R^3}\frac{d}{dt}\left(\rho_m R^3\right) + \frac{1}{R^4}\frac{d}{dt}\left(\rho_r R^4\right) = 0 \tag{10}$$

From the above equation we notice immediately that if $\dot{\rho}_v \neq 0$ at least one of the ordinary adiabatic relations $\rho_r \sim R^{-4}$, $\rho_m \sim R^{-3}$ ceases to be valid.

In view of this we consider the ansatz given by Islam [14] for the variation of density ρ as

$$\rho = \frac{A}{R^4}\sqrt{R^2 + b} \tag{11}$$

where A and b are positive constants, in presence of radiation and matter both which is also the second condition required to close the system. We can see, for small R, the term b dominates R and the variation of the energy density is R^{-4} describing the radiation era of standard scenario. Similarly, for large R, the term b is dominated and the variation of energy density is R^{-3} describing the matter dominated era. Now, from equation (6), (7), (9) and (11), we obtain

$$\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} + 4\pi GA\left(1+w\right)\frac{\sqrt{R^2+b}}{R^4} = 0 \tag{12}$$

whose first integral is given by

$$\dot{R}^2 = \alpha R^2 - k + \frac{\pi GA\left(1+w\right)}{b}\left[\begin{array}{c} \frac{\left(R^2+2b\right)\sqrt{R^2+b}}{R^2} \\ +\frac{R^2}{\sqrt{b}}\log\left(\frac{\sqrt{R^2+b}-\sqrt{b}}{R}\right)\end{array}\right],\ b \neq 0 \tag{13}$$

where α is a constant of integration. This can further be integrated to give

$$t = \int\left[\alpha R^2 - k + \frac{\pi GA\left(1+w\right)}{b}\left\{\begin{array}{c} \frac{\left(R^2+2b\right)\sqrt{R^2+b}}{R^2} \\ +\frac{R^2}{\sqrt{b}}\log\left(\frac{\sqrt{R^2+b}-\sqrt{b}}{R}\right)\end{array}\right\}\right]^{-1/2} \tag{14}$$

The expression for the vacuum energy is obtained from equations (13) and (6) as

$$\rho_v = \frac{3\alpha}{8\pi G} - A\frac{\sqrt{R^2+b}}{R^4} + \frac{3A\left(1+w\right)}{8b}\left[\begin{array}{c}\frac{\left(R^2+2b\right)\sqrt{R^2+b}}{R^4}\\ +\frac{1}{\sqrt{b}}\log\left(\frac{\sqrt{R^2+b}-\sqrt{b}}{R}\right)\end{array}\right] \tag{15}$$

The Hubble parameter H and deceleration parameter q in the model are obtained as

$$H^2 = \alpha - \frac{k}{R^2} + \frac{\pi GA\left(1+w\right)}{b}\left[\begin{array}{c}\frac{\left(R^2+2b\right)\sqrt{R^2+b}}{R^4}\\ +\frac{1}{\sqrt{b}}\log\left(\frac{\sqrt{R^2+b}-\sqrt{b}}{R}\right)\end{array}\right] \tag{16}$$

and

$$q = -1 - \frac{\frac{k}{R^2} - 4\pi GA\left(1+w\right)\frac{\sqrt{R^2+b}}{R^4}}{\alpha - \frac{k}{R^2} + \frac{\pi GA(1+w)}{b}\left[\frac{(R^2+2b)\sqrt{R^2+b}}{R^4} + \frac{1}{\sqrt{b}}\log\left(\frac{\sqrt{R^2+b}-\sqrt{b}}{R}\right)\right]} \tag{17}$$

3.1. Case-1

When $b = 0$ i.e. $\rho = (\rho_m + \rho_r) \sim \frac{1}{R^3}$ equation (12) reduces to

$$\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} + 4\pi GA\left(1+w\right)\frac{1}{R^3} = 0 \tag{18}$$

which yields

$$\dot{R}^2 = \alpha R^2 - k + \frac{8\pi GA\left(1+w\right)}{3R} \tag{19}$$

The vacuum energy, Hubble parameter and deceleration parameter are found to be

$$\rho_v = \frac{3\alpha}{8\pi G} + \frac{Aw}{R^3} \tag{20}$$

$$H^2 = \alpha - \frac{k}{R^2} + \frac{8\pi GA\left(1+w\right)}{3R^3} \tag{21}$$

and

$$q = -1 - \frac{\frac{k}{R^2} - \frac{4\pi GA(1+w)}{R^3}}{\alpha - \frac{k}{R^2} + \frac{8\pi GA(1+w)}{3R^3}} \tag{22}$$

We see that for large values of R, ρ_v approaches to a constant value $\frac{3\alpha}{8\pi G}$, H approaches to a constant value $\sqrt{\alpha}$ and q approaches to -1 describing an accelerating Universe at late times.

3.2. Case-2

For $k = 0$, $b = 0$, we have from equation (18), the explicit form of the scale factor is obtained as,

$$R(t) = \left(\frac{8\pi GA\,(1+w)}{3\alpha}\right)^{\frac{1}{3}} \sinh^{\frac{2}{3}}\left(\frac{3\sqrt{\alpha}}{2}t + t_0\right) \tag{23}$$

where t_0 is a constant of integration which may be taken as zero. The solution (23) for large values of t reduces to $R(t) \sim e^{\sqrt{\alpha}t}$, which corresponds to an empty de Sitter universe with accelerated expansion. The equation of state parameter w assumes two values $w = 1/3$ in radiation dominated era and $w = 0$ in matter dominated eras.

RD phase ($w = 1/3$)

In radiation dominated phase, the cosmological parameters are obtained as,

$$R(t) = \left(\frac{32\pi GA}{9\alpha}\right)^{\frac{1}{3}} \sinh^{\frac{2}{3}}\left(1.5\sqrt{\alpha}t\right), \tag{24}$$

$$H(t) = \sqrt{\alpha}\coth\left(1.5\sqrt{\alpha}t\right), \tag{25}$$

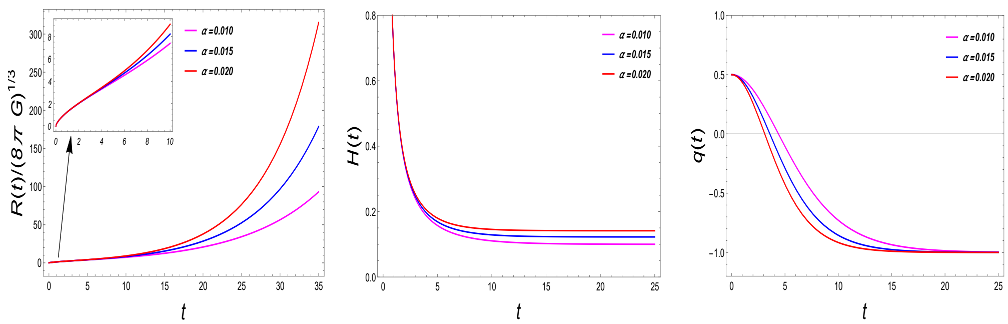
$$q(t) = -1 + \frac{3}{1+\cosh(3\sqrt{\alpha}t)} \tag{26}$$

and

$$8\pi G\rho_r(t) = \frac{9\alpha}{4\sinh^2(1.5\sqrt{\alpha}t)}, \tag{27}$$

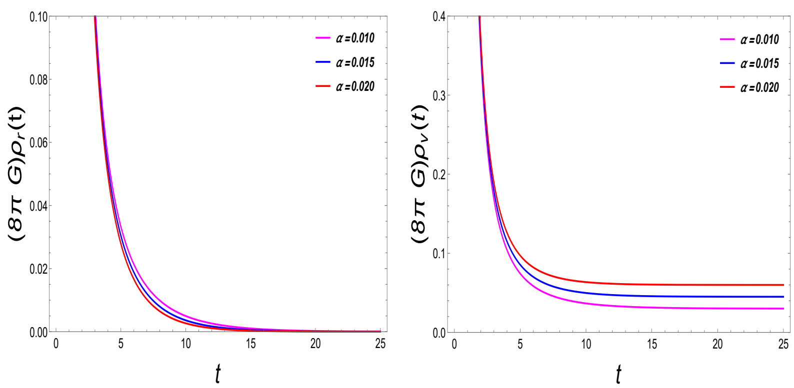
$$8\pi G\rho_v(t) = \Lambda = 3\alpha\coth^2(1.5\sqrt{\alpha}t). \tag{28}$$

We can see, in the early phase of evolution, while $t \to 0$, the scale factor $R \to 0$ implying a big bang origin of the Universe in this model. The following figures describe the evolution of these cosmological parameters for some suitable choice of the constants A and α with suitable time units.



(a) (b) (c)

FIGURE 1. The panels (a), (b) and (c) in this figure respectively show the evolution of the scale factor R, Hubble parameter H and deceleration parameter q w.r.t. cosmic time t with $A = 2$ and different values of $\alpha - 0.010$, 0.015, 0.020.



(a) (b)

FIGURE 2. The panels (a) and (b) in this figure respectively show the evolution of the scale factor radiation energy density ρ_r, vacuum energy density ρ_v w.r.t. cosmic time t with $A = 2$ and different values of $\alpha - 0.010$, 0.015, 0.020.

MD phase ($w = 0$)

In the matter dominated era, we have

$$R(t) = (8\pi G)^{\frac{1}{3}} \left(\frac{A}{3\alpha}\right)^{\frac{1}{3}} \sinh^{\frac{2}{3}}\left(1.5\sqrt{\alpha}t\right). \tag{29}$$

While studying present behavior of the Universe, it is convenient express the cosmological parameters in terms of redshift z (where $1+z=\frac{R_0}{R}$, R_0 being the present value of scale factor and is normalized to $R_0 = 1$) for which, the t-z relationship can be established as $t(z) = \frac{2}{3\sqrt{\alpha}}\sinh^{-1}\left(\sqrt{\frac{3\alpha}{8\pi GA}}(1+z)^{-\frac{3}{2}}\right)$. Other geometrical and physical parameters can be represented expressed in terms of redshift z as follows.

$$H(z) = H_0 \frac{\coth\left[\sinh^{-1}\left(M_{pl}\sqrt{\frac{3\alpha}{A}}\,(1+z)^{-\frac{3}{2}}\right)\right]}{\coth\left[\sinh^{-1}\left(M_{pl}\sqrt{\frac{3\alpha}{A}}\right)\right]}, \tag{30}$$

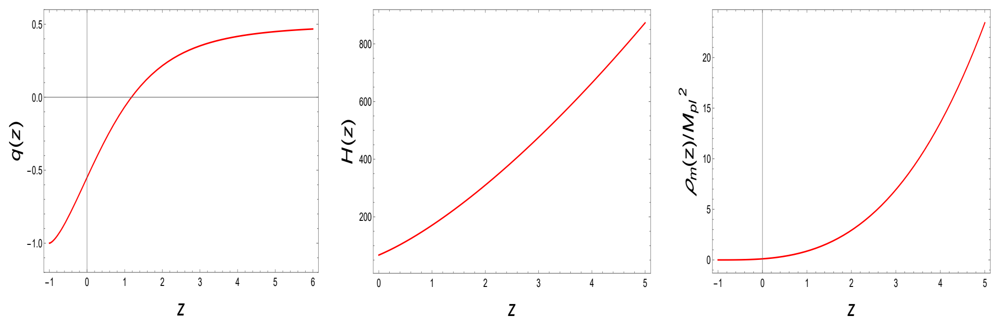
$$q(z) = -1 + \frac{3}{1+\cosh\left[\sinh^{-1}\left(M_{pl}\sqrt{\frac{3\alpha}{A}}\,(1+z)^{-\frac{3}{2}}\right)\right]} \tag{31}$$

and

$$\rho_m(z)/M_{pl}^2 = \frac{9\alpha}{4\sinh^2\left[\sinh^{-1}\left(M_{pl}\sqrt{\frac{3\alpha}{A}}\,(1+z)^{-\frac{3}{2}}\right)\right]}, \tag{32}$$

$$\rho_v(z)/M_{pl}^2 = \Lambda = 3\alpha. \tag{33}$$

Where $8\pi G = M_{pl}^{-2}$ and H_0 is the present value of the Hubble parameter ($H_0 = 67.4^{+0.5}_{-0.5}\ km/sec/Mpc$ according to Planck 2018 results [15]). The constant A must be of the order of M_{pl}^2. To get some rough estimate of the values of model parameters A and α, we can compare the value of any of the cosmological parameter at $z = 0$. We know H & q are two observable parameters and have been measured by various methods (e.g. cosmic chronometer, Baryonic Acoustic Oscillation method etc. for H) since past few years. So, we consider here the present calculated value of deceleration parameter $q_0 \approx -0.55$ as estimated by various observations for which we can get a relation from equation (31), $A \approx 0.0964286\ \alpha$ and if we choose $\alpha = 1.5$, we have $A = 0.144643$. For these values, we can plot the evolution of $q(z)$ at present era as follows.



(a) (b) (c)

FIGURE 3. The panel (a) in this figure shows the evolution of the deceleration parameter q vs. redshift z for $\alpha = 1.5$ and $A = 0.09642$ for which the present value of $q_0 = -0.55$. Panels (b) and (c) show the evolution $H(z)$ and $\rho_m(z)$ for the same values.

4. Conclusion

In this paper, we have discussed a dark energy model with a candidate of dark energy represented by the cosmological constant by constraining an ansatz as depicted in [14] which shows variation of energy density as a function of scale factor and connects the radiation and matter. The considered parametrization of the energy density (ρ) of matter contains two model parameters. The model reduces to the Einstein de-Sitter for vary large value of the scale factor with negative value of the deceleration parameter showing late time acceleration. Further, for some particular values of the model parameters, we have two cases. For one case, we have discussed a particular model in some details and the evolution of the cosmological parameters are shown through graphical representations. The models obtained here show acceleration at late-times which is consistent with the present observation.

Acknowledgement

One of the author SKJP dedicate this paper to his reverend PhD guide and supervisor Prof. Abdussattar.

REFERENCES

[1] Perlmutter, S. et al.: Astrophys. J., 517 (1999), 565.
[2] Riess, A. et al.: Astrophys. J., 117 (1999), 707.

[3] Turner, M. S. and Huterer, D.: J. Phys. Soc. Jap., 76 (2007), 111015.
[4] Copeland, E. J., Sami, M. and Tsujikawa, S.: Int. J. Mod. Phys. D, 15 (2006), 1753.
[5] Bamba, K. et al.: Astrophys. Space Sci., 342 (2012), 155.
[6] Zlatev, I., Wang, L. M. and Steinhardt, P. J.: Phys. Rev. Lett., 82 (1999), 896.
[7] Brax, P. and Martin, J.: Phys. Rev. D, 61 (2000), 103502.
[8] Barreiro, T., Copeland, E. J. and Nunes, N. J.: Phys. Rev. D, 61 (2000), 127301.
[9] Armendariz-Picon, C., Damour, T. and Mukhanov, V.: Phys. Lett. B, 458 (1999), 219.
[10] Chiba, T., Okabe, T. and Yamaguchi, M.: Phys. Rev. D, 62 (2000), 023511.
[11] Armendariz-Picon, C., Mukhanov, V. and Steinhardt, P. J.: Phys. Rev. Lett., 85 (2000), 4438.
[12] Pacif, S. K. J. et al.: Int. J. Geom. Meth. Mod. Phys., 14(7) (2017), 1750111.
[13] Vishwakarma, R. G. and Narlikar, J. V.: J. Astrophys. Astron., 28 (2007), 17 .
[14] Islam, J. N.: *An Introduction to Mathematical Cosmology* (Cambridge University Press, Cambridge, England) (1992).
[15] Aghanim, N. et al.: arXiv:1807.06209v2 (2018).