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Constraints on Background Torsion through Nonminimal coupling of Torsion to Electromagnetism[1]

A.R. Prasanna, Subhendra Mohanty*, and Moumita Das***

*Physical Research Laboratory, Ahmedabad, India

**L J Institute of Computer Applications, Ahmedabad, India
E-mail: aragam@gmail.com

Abstract

The role of curvature and torsion in the formulation of geometrodynamics of matter and electromagnetism is discussed and a case study of minimal coupling of electromagnetism to torsion background has been made.

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1. Introduction

The most interesting aspect of our Universe is that as the combination of Space, Time and Matter, it can be described by Geometry, as shown by Einstein through his Principle of General Relativity, which today is the basis of any successful Cosmological model. Differential Geometry (which was the topic of research of Prof Hussain) which governs the manifold structure of spacetime, deals mainly with two major attributes 'metric' and the 'connection'. Through Einstein's equations, while metric represents the 'Potential', the connection represents the 'Force', and the two together govern the Physics of the Universe. It is well understood that the metric or the distance function, which comes out as a solution of Einstein's equations, for a given distribution of matter and energy, describes the geometry and thus the theory is called as the geometrodynamics. While Torsion did not find any place in Einstein's original theory, it got its due in the formulation of Cartan (1922) [1], who identified it as being the geometrical representation of Spin, similar to the role of curvature in representing the Energy and Momentum. First formal presentation of this theory, known today as Einstein-Cartan theory, was given by Trautman (1972)[2] and the mathematical structures of

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torsion theories is elucidated by Hehl et al.[3]. Prasanna [4] has given a simple treatment of Einstein-Cartan equations while looking for its solutions both for matter and electromagnetic fields. As the contribution of torsion appears with the quadratic coupling constant of gravity, it becomes extremely difficult to associate any directly observable features with it. However, due to its nature of antisymmetry in two indices, one of the natural ways to look for its effects appears to be , through coupling it with the electromagnetic field tensor F which is also antisymmetric. There have been several discussions in this context and particularly associated with the fact that in the last two decades there has been a large amount of data gathered regarding various facets of the cosmic microwave background radiation the relic of the early universe. An earlier study of Prasanna and Mohanty [5], regarding the effect of background torsion on the propagation of electromagnetic waves, with minimal coupling of Torsion to electromagnetism, had yielded an upper bound on its presence. Comprehensive review articles have been written by R.T.Hammond [6] and I.L.Shapiro [7], in 2002 on the various aspects of the torsion physics both for classical and quantum aspects ,as well as pointing out the role of torsion in string theories.

2. Torsion and birefringence

In general relativity, with Riemannian background, the connection is defined through the covariant derivatives,

$$\nabla_\mu V_\nu = \partial_\mu V_\nu + \Gamma^\alpha_{\mu\nu} V_\alpha \tag{1}$$

where $\Gamma^\alpha_{\mu\nu}$ is symmetric in lower indices and is known as Levi-Civita connection. On the other hand, in a general differential manifold, the connection

$$\Gamma^\alpha_{\mu\nu} = \{^\alpha{}_{\mu\nu}\} + T^\alpha_{\mu\nu}, \tag{2}$$

is asymmetric with its symmetric part $\{^\alpha{}_{\mu\nu}\}$ being the Levi-Civita part, also known as the Christoffel symbol and the part represented by $T^\alpha_{\mu\nu}$, is called Torsion which is antisymmetric in μ and ν. For metric compatible connection, where the metric $g_{\mu\nu}$ is a covariant constant,

$$\{^\alpha{}_{\mu\nu}\} = \frac{1}{2} g^{\alpha\lambda}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu-}), T^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu}. \tag{3}$$

In this discussion, we shall consider the generalized connection restricting ourselves to metric theories such that we still have $\nabla_\mu g_{\alpha\beta} = 0$, (metric a covariant constant) and use the conventions $\eta =$ diag$(1, -1, -1, -1)$, for the Minkowski metric and $\varepsilon_{123} = 1$, $\varepsilon^{123} = -1$, for the permutation symbol.

Coupling torsion to electromagnetism in a gauge-invariant fashion was first carried out by Novello [8], De Sabbata and Gasperini [9] and Duncan, Kaloper and Olive [10]. They pointed out that if the dual of the torsion tensor, $T^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} T_{\nu\alpha\beta}$ is the divergence of a scalar, $T_\mu = \partial_\mu \phi$, then it can be coupled to electromagnetic interactions in a gauge invariant way by the interaction $T_\mu A_\nu {}^\sim F^{\mu\nu} = \phi F_{\lambda\mu} {}^\sim F^{\mu\nu}$.

A propagating scalar give rise to long range forces which can be constrained by the observations of energy loss of Hulse-Taylor binary pulsar [11]. A cosmological scalar field which couples to electromagnetism causes birefringence of electromagnetic waves causing polarization of radio signals from distant galaxies as pointed out by Harari and Sikivie [12]. Carroll and Field [13] put strong constraints on the presence of cosmological background torsion from the polarization of radio galaxies. Hammond [14] introduced an antisymmetric two-index torsion potential $\Psi_{\mu\nu}$ related to the torsion tensor by $T_{\alpha\mu\nu} = \partial_{[\alpha\Psi_{\mu\nu}]}$ and coupled it to electromagnetism through an interaction of the form $F^{\mu\nu}\Psi_{\mu\nu}$. This interaction is similar to the kinetic mixing between photons and para-photons studied by Masso and Redondo [15, 16]. If the scalar-torsion fields have heavier mass, then there are laboratory constraints on their coupling to electromagnetism as shown in the optical polarization of lasers in external magnetic field [17, 18] (as proposed in [19, 20]) and in long range non-Newtonian forces [21]. Constraints on heavy propagating torsion which may be produced in accelerators or in supernovae have been surveyed in [13], [22]. The coupling of background torsion on spin of fermions has been constrained from precision experiments [23].

In this paper we study non-minimal coupling of a cosmological background torsion field to electromagnetic field of the form $\xi_1 T^{\alpha\lambda}{}_{\rho} F_{\alpha\nu}\partial_\lambda {}^{\sim}F^{\rho\nu}$ and $\xi_2 T^{\sigma\gamma}{}_{\delta} F_{\sigma\nu}\partial_\gamma {}^{\sim}F^{\delta\nu}$. We find that the first type of coupling causes birefringence in the electromagnetic waves propagating through a background torsion field over cosmological distances. The ξ_1 type of coupling is similar in structure to the form $(1/EP)n^\alpha F_{\alpha\sigma} n^\gamma \partial_\gamma (n_\beta {}^{\sim}F^{\alpha\beta}$ proposed by Myers and Pospelov[24] motivated by quantum gravity arguments. A detailed study of the birefringence arising from Myers-Pospelov interaction on the CMB polarization was done by Gubitosi et al. [25].

For the second type of coupling, the transverse modes of electromagnetic waves propagate along null geodesics despite the presence of torsion coupling. Therefore, the ξ_2 type of coupling has no effect on the wave propagation. These types of couplings are also similar in form to the phenomenological Lorentz violating couplings of electromagnetism studied in Kostelecky and Mewes [26] who proposed that such couplings will mix the E and B mode CMB polarization due to birefringence. Feng et al.[27] analyzed the polarization data from WMAP and BOOMERanG and concluded that the data indicates a non zero value of the angle of rotation of the plane of polarization. Gubitosi et al. [25] put a limit on the angle of rotation of the plane of polarization of CMB signal at $\alpha = (-2.4 \pm 1.9)^\circ$ using the data from WMAP and BOOMERanG, .We use this result of the angle of rotation of CMB to see whether we can put limits on the non-minimal coupling of torsion with electromagnetism.

3. Non minimal coupling of Torsion with Electrodynamics

We start with the Lagrangian for the electromagnetic field on a manifold with torsion but no spatial-curvature. The metric at cosmological scales is a conformally flat Robertson-Walker metric,

$$ds^2 = a(\tau)^2(d\tau^2 - (dx^2 + dy^2 + dz^2)) = a(\tau)^2 \eta_{\mu\nu} dx^\mu dx^\nu \tag{4}$$

where $a(\eta)$ is the scale factor of the expanding universe and $\tau = \int dt/a(t)$ is the conformal time. We consider the two types of non-minimal couplings of torsion with electromagnetism as described by,

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \xi_1 T^{\alpha\lambda}{}_{\rho} F_{\alpha\nu}\partial_\lambda {}^{\sim}F^{\rho\nu} + \xi_2 T^{\sigma\gamma}{}_{\delta} F_{\sigma\nu}\partial_\gamma F^{\delta\nu}. \tag{5}$$

We will assume that the space-time variation of the background tensor is on a larger scale than the wavelength of the propagating photons, and thus while deriving the equations of motion we will use the simplifying assumption, $\partial\mu T_{\delta}^{\sigma\gamma} \nabla k\mu T_{\delta}^{\sigma\gamma}$ where $k\mu$ is the four-momentum of the propagating photon. We will consider the two coupling terms separately. when $\xi_1 \neq 0$ and $\xi_2 = 0$.
Using the Euler-Lagrangian equations

$$\partial\mu[\partial L/\partial(\partial\mu A\nu)] - \partial L/\partial A\nu = 0 \tag{6}$$

the equation of motion will be,

$$\partial i E^i = \xi 1 T^{\alpha\lambda} i \partial\alpha\partial\lambda B^i \tag{7}$$

$$\partial 0 E^j + \partial i \varepsilon^{0ijk} Bk = \xi 1 \partial\alpha\partial\lambda[-T^{\alpha\lambda}0 B^j + T^{\alpha\lambda} i \varepsilon^{0ijk} Ek] \tag{8}$$

and from the Maxwell's equation for $F\mu\nu$,

$$\partial k \varepsilon^{0kijl} E1 - \partial 0 B^j = 0 \tag{9}$$

$$\partial_j B^j = 0 \tag{10}$$

Using eq. (9), eq. (8) can be written as,

$$-\partial^0\partial 0 E^j - \partial^m(\partial_m E^j - \partial^j E_m) = -\xi 1 \partial\alpha\partial\lambda[T^{\alpha\lambda}0\varepsilon^{0mjn}\partial m En - \Gamma^{\alpha\lambda} i \varepsilon^{0ijk}\partial^0 Ek] \tag{11}$$

which is the equation of motion for the electric field. If we consider the plane wave solution where, the electric field can be written as, $Em(x) = Em(k)\ exp\,(-ik_\mu x^\mu)$ eq. (11) reduces to,

$$k_0 k^0 E^j + k^m(kmE^j - k^j Em) = -i\xi 1 k\alpha k\lambda[T^{\alpha\lambda}0\varepsilon^{0mjn} kmEn + \Gamma^{\alpha\lambda} i\varepsilon^{0ijk} k^0 Ek] \tag{12}$$

Choosing Z-axis along the direction of propagation i.e. $k=(\omega, 0, 0, p)$, the three equations of motion given by the eq. (12) take the form,

$$\begin{aligned}(\omega^2 - p^2)E_1 &= 2i\xi_1 T_3 p^3 E_2\\ (\omega^2 - p^2)E_2 &= -2i\xi_1 T_3 p^3 E_1\\ \omega^2 E_3 &= i\xi_1(T_2E_1 - T_1E_2)p^3\end{aligned} \tag{13}$$

where

$$\begin{aligned}T_1 &\equiv T_1^{00} + T_1^{03} + T_1^{30} + T_1^{33}\\ T_2 &\equiv T_2^{00} + T_2^{03} + T_2^{30} + T_2^{33}\\ T_3 &\equiv T_3^{00} + T_3^{03}\end{aligned} \tag{14}$$

From (13) one can get the equation of motion for the transverse modes, and thus the equations for the right and left circularly polarized fields, $E\pm = E_1 e^x \pm E_2 e^y$, to be;

$$(\omega^2 - p^2)(E_1 \pm iE_2) \pm 2\xi_1 p^3 T_3 (E_1 \pm iE_2) = 0 \tag{15}$$

From the equations of motion for the modes E_1 and E_2, one can get the dispersion relation

$$(\omega\pm = p(1 \pm \xi_1 p T_3) \tag{16}$$

The rotation of the plane of polarization can be written in terms of the difference of the $\omega\pm$, as,

$$\alpha = (\omega^- - \omega^+)t = 2\xi_1 p^2 T_3 t \tag{17}$$

where t is the propagation time, as given by:

$$t = (1/H^\circ) \int_0^z (1+z)/\sqrt{(\Omega_m(1+z)^3 + \Omega_\Lambda)} dz. \tag{18}$$

One notices that only the combination, $T_3 = T_0^{03} - T^3 3)_0$, enters the equation because of the choice of z — axis along the direction of propagation.

Experimental constraints on α have been put at (-2.4 ± 1.9) degrees (Gubitosi et al. [20]), from the observation of CMB polarization through WMAP and BOOMERanG. One can make use of these constraints to put bounds on the torsion coupling $\xi_1 T_3$,. using in the relation (18), the values $H^\circ = 72(Km/s)/Mpc$, $z = 1100$, $p = 100GHz, , \Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, to get the value of 't'.

One finds

$$\xi_1 T_3 = (-3.35 \pm 2.65) \times 10^{-22} GeV^{-1} \tag{19}$$

which is comparable with the limits on Lorentz violating interaction of electromagnetism as shown earlier.

4. Discussions

Having described briefly the roles of Curvature and torsion in the formulation of geometrodynamics of matter and electromagnetism, We have presented a case study of non-minimal coupling of electromagnetism to torsion background of the form $\xi_1 T^{\alpha\lambda}{}_\rho F_{\alpha\nu}(\partial_\lambda {}^\sim F^{\rho\nu})$ and $\xi_2 T^{\sigma\gamma}{}_\delta F_{\sigma\nu}(\partial_\gamma F^{\delta\nu})$. It is shown that the first type ,with $(TF^\sim F)$ does have an effect similar to what was obtained by Myers and Pospelov, for a nonminimal coupling of the form $\xi_1 n^\alpha F_{\alpha\sigma} n^\gamma \partial_\gamma (n_\beta {}^\sim F^{\alpha\beta}$, where the vector n^α is some fixed Lorentz invariance violating object. It appears that the background Torsion also acts like such a vector field. Hence one could deduce some constraint on its effect on the CMB, thus making it possible to constrain the torsion magnitude through the observation of CMB polarisation.

On the other hand it was realized that, the second type of interaction with (TF^2), irrespective of its magnitude seems to cause no birefringence as the wave propagate on null geodesics and thus CMB cannot offer any constraint on its effect. The fact that only the ξ_1 term has the Levi-Civita tensor which is odd under parity could be the cause that this coupling differentiates between the left and the right circular polarization leading to birefringence.

It is our pleasure to dedicate this article to the memory of Professor Izhar Husain, with whom I (A.R.Prasanna) had the privilege to associate in connection with UGC committees and other activities of IAGRG. Professor Husain, apart from being a dedicated Relativist and Geometer, was a very active academic who contributed in building up the activities of IMS and of the Tensor society.

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