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Stellar Collapse with Radiation on Inhomogeneous Space Time Background

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Abstract
The collapse of an inhomogeneous spherical distribution of matter in the presence of outflow of radiation is examined on the background of the space time obtained by introducing an inhomogeneous perturbation in the Robertson Walker space time so that the physical 3-space has geometry of 3-spheroid, 3-pseudospheroid or 3-paraboloid. The set up is used to discuss the collapse of a static compact star with anisotropic matter or perfect fluid, when its equilibrium is lost. The homogeneous dust collapse model on FRW background and its generalizations describing inhomogeneous collapse of a fluid in the presence of radiation on the background of spheroidal space time, follow on appropriate choice of parameters.

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1. Introduction

The problem of obtaining a realistic description of the interior space time of a stellar body collapsing under its own gravity accompanied with presence of radiation and predicting its future evolution within the framework of Einstein's Theory of Relativity has received considerable attention. The inward gravitational attraction of material content in a star is counterbalanced by the repulsive fluid pressure the star, maintaining a state of stellar equilibrium. When this equilibrium is lost, the stellar structure begins to collapse under self gravity and the thermal processes which are ignorable on a short time scale begin to play a significant role on astronomical scales. The gravitational collapse of spherical dust cloud for adiabatic flow under simplifying conditions was first studied by Oppenheimer and Snyder (1939). Later Vaidya (1965, 1966) Lindquist et al (1965) studied homogeneous collapsing spherical distributions of dust in the presence of outgoing null electromagnetic radiation on the background of FRW space times. Misner and Sharp (1965), Misner (1965) studied the relationship of the outgoing radiation with the collapsing matter in the interior. Vaidya and Patel (1996) extended these ideas by considering inhomogeneous collapsing fluid distributions in presence of null electromagnetic radiation and gave an approximation procedure leading to an inhomogeneous generalization of Oppenheimer-Snyder model of collapse with outflow of electromagnetic radiation.

Santos (1985) proposed a procedure to carry out more realistic analysis of the collapse of radiating spherical bodies undergoing non-adiabatic collapse under self gravity using relativistic models of shear-free collapsing fluids with radial heat flow first proposed by Glass (1981). Santos and co workers (de Oliveira et al 1985, 1986, 1987, 1988) used these ideas and studied various aspects associated with collapse of a radiating star. These studies showed how the fluid pressure on the boundary of the star is related with outward heat flux.

The relativistic models of super dense stars such as neutron stars, strange stars, quark stars are usually studied by integrating numerically the appropriate set of Einsteins field equations (EFEs) on the basis of an already accomplished equation of state (EOS) for their matter content. The precise nature of the behavior of the content of compact stars such as mentioned above is not known with certainty implying that there is no reliable information about the EOS of matter for such stars. Subsequently the highly non-linear system of relevant EFEs governing the structure of the interior space time are solved by stipulating assumptions of general nature for its matter content. The approach followed by Vaidya -Tikekar (1982), Tikekar (1990) of prescribing suitable compact geometry to the interior physical space of the relativistic stars is found to be very useful for obtaining tractable models of such stars. Various closed form solutions of EFEs in the context of the 3-spheroidal space-time geometry have been investigated by Maharaj and Leach (1996), Mukherjee et al, (1997), Gupta and Jassim (2000). The physical plausibility and stability of the models of Vaidya-Tikekar class have been extensively examined by Knutsen (1988, 1989). The usefulness of other similar geometric ansatz has also been indicated in this context (Tikekar and Thomas 1998, 1999, Tikekar and Jotania 2005, 2006, 2007). An important feature of this approach is that it admits existence of a relativistic compact star with mass exceeding the limiting value of 3.20 solar mass, for maximum mass of neutron stars which was obtained by Rhodes and Ruffini (1974) through general considerations. It is expected that such massive compact stars may be unstable and may collapse when their equilibrium is disturbed. In the following sections certain aspects of the collapse of such massive super dense stars respectively in the presence of (i) non adiabatic dissipative forces admitting presence of heat flow and (ii) electromagnetic radiation have been discussed on the background of inhomogeneous space times obtained by introducing perturbations in the standard RW space time.

2. Non adiabatic collapse with heat flux

Following the approach of Tikekar and Patel (1992) we assume that the interior space-time of a shear-free, spherically symmetric collapsing star is adequately described by the metric

$$ds^2 = -e^{\beta(t)}[e^{2\mu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta dp^2)] + e^{2\gamma(r,t)}dt^2, \qquad (1)$$

where $e^{2\mu(r)} = \frac{1-K(r/R^2}{1-k(r/R^2}$.

The associated physical 3-space, obtained as t =constant section, of the space-time is a maximally symmetric homogeneous and isotropic 3-space when the parameter $K = 0$. The parameter K is a measure of the in-homogeneity or of the departure of the geometry of physical 3-space from spherical, flat or hyperbolic nature, R denoting the Gaussian curvature of the background 3-space. The physical 3-space has the geometry of

(i) A 3-sphere of radius R, when $k = 1$, $K = 0$,

(ii) A 3-spheroid if $k = 1$, $K < 1$,

(iii) A 3-pseudo-spheroid if $k = 1$, $K < 1$,

(iv) A 3-paraboloid if $k = 0$, $K = 1$.

The interior of the collapsing star is considered in the form of non-adiabatic matter, a fluid with anisotropic pressure accompanied with heat flow along radial outward direction described by energy momentum (EM) tensor

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} + \pi_{ij} + q_i u_j + q_j u_i \qquad (2)$$

Here ρ, p and, $u^j(= e^{-\gamma}\delta^j_4)$ respectively represent matter density, fluid pressure and unit 4-flow field of matter. Further $q^j(= q\delta^i_1)$ denotes space like heat flux vector orthogonal to u^i and

$$\pi_{ij} = \sqrt{3}S(r,t)[C_i C_j + (1/3)(g_{ij} - u_i u_j)]C^i = e^{-\mu+(\beta/2)}\delta^i_1, \quad i = 1,2,3 \qquad (3)$$

are the components of anisotropic pressure tensor with S(r,t) denoting its magnitude. EFEs in system of units rendering $8\pi G = c = 1$, is the system of following four equations:

$$\rho = \frac{3}{4}e^{-2\gamma}\beta^2 + e^{-\beta}[\frac{1}{r^2} - e^{-2\mu}(\frac{1}{r^2} - \frac{2\mu^i}{r})], \qquad (4)$$

$$p_r = -e^{-2\gamma}[\beta''2 + \frac{3}{4}\beta^2 - \gamma\beta'] - \frac{e^{-\beta}}{r^2} + e^{-2\mu+\beta}[\frac{1}{r^2} + \frac{2\gamma'}{r}], \qquad (5)$$

$$p_T = -e^{-2\gamma}[\beta''2 + \frac{3}{4}\beta^2 - \gamma'\beta'] - \frac{e^{-\beta}}{r^2} + e^{-2\mu+\beta}[\gamma'' + \gamma'^2 - \gamma'\mu' - \frac{\mu'}{r} + \frac{\gamma'}{r}], \qquad (6)$$

$$q = -e^{-(\gamma+2\mu+\beta)}\beta\gamma'. \qquad (7)$$

Here p_r and p_T denote fluid pressures along - radial and transverse to radial-directions. The anisotropy parameter i.e. magnitude of stress tensor has the following explicit expression

$$\sqrt{3}S = p_r - p_T \tag{8}$$

If the interior matter of the star is a perfect fluid in equilibrium, pressure isotropy condition demands $p_r - p_T = 0$.

The space time in the exterior of the star will be described by the Vaidya (1951) metric

$$ds^2 = -(1 - \frac{2m(v)}{r})dv^2 - 2dvdr + r^2(d\theta^2 + \sin^2\theta d\psi^2). \tag{9}$$

Here v is the retarded time and $m(v)$ denotes the total mass within spherical region of radius r.

The matching of the metrics (1) and (9) across the shrinking boundary the 2-hyper surface Σ, links, the radial pressure on boundary $(p_r)_\Sigma$ with the heat flux q_Σ across Σ and the relation has the explicit form (de Oliveira et al 1985)

$$(p_r)_\Sigma = (qe^{\mu}e^{\beta/2})_\Sigma. \tag{10}$$

Equation (4) on integration has been known to lead to

$$2m(r,t) = re^{\beta/2}(1 - e^{-2\mu} + \frac{1}{4}\beta^2\gamma^2 e^{(\beta-2\gamma)}, \tag{11}$$

where $m(r,t)$ denotes the total mass enclosed within a spherical region of radius $r < r_\Sigma$-the radius of shrinking spherical boundary. The function $m(v)$ in (9) will be

$$2m(v) = 2m(r_\Sigma, t). \tag{12}$$

The interior space time of the spherical star before the collapse begins is described by the metric

$$ds^2 = e^{2\gamma(r)}dt^2 - e^{2\mu(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\psi^2), \;\; e^{2\mu(r)} = \frac{1 - K(r/R)^2}{(1 - 1 - k(r/R)^2}. \tag{13}$$

The space time in the exterior region before the collapse begins is described by the Schwarzschild exterior metric

$$ds^2 = (1 - \frac{2m}{r})dt^2 - (1 - \frac{2m}{r})^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\psi^2). \tag{14}$$

Here $m = MG/c^2$ is a constant related with the mass M of the star. When its equilibrium is lost the star begins to collapse from its equilibrium state. The interior and exterior space times are then respectively described by (1) and (9). The dynamical equations governing the collapse (3), (4), (6), and (7) can be expressed in the following more convenient form:

$$\rho = \frac{3}{4}e^{-2\gamma}\beta^2 + e^{-\beta}\rho_0, \tag{15}$$

$$p_r = -e^{-2\gamma}[\beta''2 + \frac{3}{4}\beta^2 - \gamma\beta'] + e^{-\beta}(p_r)_0, \tag{16a}$$

$$p_T = p_r + e^{-\beta}(p_T - p_r)_0 \tag{16b}$$

$$q = -e^{-(\gamma+2\mu+\beta)}\beta\gamma'. \tag{17}$$

In above the suffix '0' refers to state of star model before the collapse sets in. It should be noted that Equation (16) implies that if the matter distribution before the collapse begins has anisotropic pressure, the pressure anisotropy will prevail throughout as the collapse continues. If the collapsing stellar matter is a perfect fluid before the beginning of its collapse, the pressure isotropy will prevail and will continue to prevail later as the collapse proceeds.

The boundary condition (10) leads to differential equation determining $\beta(t)$

$$e^{\beta}(\beta'' + \frac{3}{4}\beta^2 - \gamma'\beta')e^{\gamma-\mu+(\beta/2)}\beta\gamma']_{\Sigma} = 0. \tag{18}$$

The evolution of the collapse of model- star as governed by (18) with interior matter in the form of perfect fluid was examined stipulating $\gamma = \gamma(r)$ in (1). Equation (18) is found to admit the general solution

$$t = [\{(e^{\beta/2}/2) + e^{\beta/4} + ln(1 - e^{\beta/4})\}/b] \tag{19}$$

where

$$b = [e^{\gamma-\mu+(\beta/2)}\gamma']_{\Sigma}. \tag{20}$$

At $t = 0$, $e^{\beta} = 0$ and as $t \to -\infty$, $e^{\beta} \to 1$. Let $a(t) = (re^{\beta})_{\Sigma}$ denote the commoving 2-boundary surface of the collapsing model star and $a(0) = a_0$ when collapse begins. The mass function

$$m(v) = [m_0 + r^3 e^{-2\gamma-\beta}(d\beta/dt)^2/8]. \tag{21}$$

When collapse begins let $m_0 = a_0(1e^{2\mu(a)})_0/2$ denote the mass of the star. Let the collapse begin at $t = \infty$ and $e^{\beta} = 1$ then. The shrinking 2-boundary surface Σ crosses into horizon when $2m(v) = a(t)$. The time of formation of a black hole $t = t_{BH}$ when this happens and the radius a_{BH} of the black hole formed, have explicit expressions

$$t_{BH} = [\{(e^{\beta/2}/2) + e^{\beta/4} + ln(1 - e^{\beta/4})\}/b]_{t_{BH}}, \tag{22}$$

$$a_{BH} = 2[2b^2 r^{3e^{-2\gamma}}(1 - e^{\beta}/4)^2 + m_0 e^{\beta}]_{t_{BH}}. \tag{23}$$

The time of formation of black hole and the radius of the black hole formed in this set up are observed to depend on the parameter b, which is determined by the initial static configuration of the model star, from which the collapse begins. One can consider a specific model describing the initially static configuration and then estimate the horizon formation time as its collapse proceeds.

The description of a star in equilibrium based on our ansatz is characterized by two parameters K, R for all k, and the two integration constants A, B. These quantities are related with the physical parameters of total mass m_0, boundary radius a_0 and central density ρ_0 of the model star through the boundary conditions relating interior space time (13) with the appropriate Schwarzschild exterior metric (14). The parameter K describing departure from spherical geometry determines the EOS of matter content of the star. The stellar structure of the model will have mass

$$m_0 = \frac{(k - K))a_0)^2}{2R^2[1 - K(a_0/R)^2]} \tag{24}$$

The configurations for different k, K and R obtained for star models with specific values of physical parameters ρ_a and density variation parameter λ ($\equiv$ ratio of surface density with central density of star-matter before the beginning of collapse) have been found to be physically plausible. The ansatz leads to models, of stellar configurations of given mass and size with different EOS corresponding to different values of density variation-parameter λ. It also allows possibilities of considering core-envelope types of stellar configurations of same mass and size having different values of λ in the core and enveloping regions. In this approach prescribing density variation parameter is equivalent to prescribing EOS of matter. It is determined by EFEs.

We studied the evolution of collapse of model stars having the same mass and size at the time when their collapse begins but having different geometries for their respective physical 3-spaces implying different EOS of matter, including core envelope kind of configurations both analytically and using numerical procedures. The analysis strongly suggests that the EOS of the interior matter which in this set up depends on the geometry of the interior 3-space of the stellar configuration does not have any impact on the evolution of collapse when it is accompanied with heat flow. It seems to be influenced by the mass and the compactness of the configuration when the collapse sets in (Tikekar and Sharma 2011).

3. Collapse with electromagnetic radiation

To examine the evolution of the collapse of model star matter in the presence of electromagnetic radiation we couch the space time metric in the following form

If $R(t) = R$, a constant and $\gamma(r,t) = \gamma(r)$, by setting (25) $r' = Rr$, (25) can be transformed into form of (1) with $\beta = 0$. The metric form (25) is suitable to study inhomogeneous collapse of mater in the presence of electromagnetic radiation on the background space time, which is deviation from the standard Robertson-Walker space time.

Oppenheimer and Snyder (1939) studied collapse of dust and Vaidya (1965) studied collapse of dust with radiation on the background of metric (25) with $k = 1$, $\gamma = 0$, $K = 0$. Vaidya and Patel (1996) studied collapse of fluid in presence of radiation on background of (25) with $\gamma = 0$, $k = 1$ and gave an approximation procedure to get a generalization of Oppenheimer-Snyder model of collapse with outflow of radiation. We have obtained procedure to describe collapse of an inhomogeneous distribution of matter with radiation flowing along outward radial direction, on the background of the space times of (25) with $\gamma \neq 0$.

The expression for EM tensor corresponding to a fluid distribution of matter density ρ and fluid pressure p moving along radial direction accompanied with a flow of radiation of density σ along radial direction is

$$8\pi T = (\rho + p)^{\nu'} \nu_j - p\delta^i_j + \sigma w' w_j, \;\; \nu^j \nu_j = 1, \;\; w^j w_j = 0, \;\; \nu^j w_j = 1. \tag{26}$$

The components of the four velocities $v^i = (v^1, 0, 0, v^4)$ of the fluid and $w^j = (w^1, 0, 0, w^4)$ of the radiation are explicitly related by relations

$$-e^{\alpha}(v_j)^2 + e^{\nu}(\nu^4)^2 = 1, \quad - e^{\alpha}(w^1)^2 + e^{\nu}(w^4)^2 = 0 \tag{27}$$

$$-e^{\alpha/2}(v^j w^1)^2 + e^{\nu}(\nu^4 w^4)^2 = 1, \;\; \text{with} \;\; e^{\alpha} = \frac{R^2(1 - Kr^2)}{(1 - kr^2)}.$$

In view of the relations in (27) we write

$$\begin{aligned} e^{\alpha/2}\nu^j &= -\sinh\beta, \quad e^{\nu/2}\nu^4 = \cosh\beta \\ w^1 e^{\alpha/2} &= w^4 e^{\nu/2}\cosh\beta - \sinh\beta, \end{aligned} \tag{28}$$

where $\beta = \beta(r, t)$ is a function to be determined.

These relations imply that the EM Tensor for matter with radiation flowing along radial direction will have components given by

$$T^1_1 = -[(\rho + p)\sinh^2\beta + p + \sigma(\cosh\beta - \sinh\beta)^2] \tag{29}$$

$$T^2_2 = -p = T^1_1 \tag{30}$$

$$T^4_4 = [(\rho + p)\cosh^2\beta - p + \sigma(\cosh\beta - \sinh\beta)^2] \tag{31}$$

$$T^1_4 = e^{(\nu-\alpha)/2}[-(\rho + p)\cosh\beta\sinh\beta + \sigma(\cosh\beta - \sinh\beta)] \tag{32}$$

$$T^4_4 = e^{(\alpha-\nu)/2}[(\rho + p)\cosh\beta\sinh\beta - \sigma(\cosh\beta - \sinh\beta)]. \tag{33}$$

Einsteins field equations in system of units rendering $G = c = 1$, then leads to the following system of four non-trivial equations sufficient to determine the four physical parameters $\rho,\ p,\ \sigma$ and the parameter β:

$$8\pi T_1^1 = -\frac{(1-kr^2)}{R^2(1-Kr^2)}\{(\gamma'/r) - \frac{(k-K)}{(1-kr^2)}\} + e^{-\nu}\{2(\ddot{R}/\dot{R}) + (\dot{R}/R)^2 - (\dot{R}\dot{\gamma}/R)\}, \tag{34}$$

$$8\pi T_2^2 = 8\pi T_3^2 = -\frac{(1-kr^2)}{R^2(1-Kr^2)}\{(\gamma''/r) + (\gamma'/r)^2 + (\gamma'/2r)\} + \frac{(k-K)[1+(r\gamma'/2)]}{R^2(1-Kr^2)}$$
$$+e^{-\nu}\{2(\ddot{R}/R) + (\ddot{R}/R)^2 - (\dot{R}\dot{\gamma}/R)\}, \tag{35}$$

$$8\pi T_4^4 = \frac{(k-K)(3-Kr^2)}{R^2(1-Kr^2)^2} + 3e^{-\nu}(2\ddot{R}/R)^2, \tag{36}$$

$$8\pi T_4^1 = \frac{(1-kr^2)\gamma'}{R^2(1-Kr^2)^2}(\ddot{R}/R)^2, \tag{37}$$

The relation $T_4^4 + T_1^1 - T_2^2 = \rho$, implies

$$8\pi\rho = \frac{(1-kr^2)}{R^2(1-Kr^2)^2}\{(\gamma''/2) + (\gamma'/2)^2 + (\gamma'/2r)\}$$
$$+\frac{(1-kr^2)(1-Kr)\gamma' + (1/2)(k-K)(5-4Kr^2)}{R^2(1-Kr^2)} + 3(\dot{R}/R)^2 e^{-\nu} \tag{38}$$

Further it follows that $T_1^1 - T_2^2 < 0$ and EFEs lead to the differential equation

$$\frac{(1-kr^2)}{R^2(1-Kr^2)^2}\{(\gamma''/2) + (\gamma'/2)^2 - (\gamma'/2r)\} - \frac{(k-K)[(r\gamma'/2) + Kr^2])}{R^2(1-Kr^2)^2} = -\xi^2 \tag{39}$$

It is possible to specify $\xi = \xi(r,t)$ suitably so that the above equation admits exact solutions. Further we have the relations

$$T_4^4 - T_2^2 = \frac{(1-kr^2)}{R^2(1-Kr^2)^2}\{(\gamma''/2) + (\gamma'/2)^2 - (\gamma'/2r)\} - \frac{(k-K)[Kr^2 + (r\gamma'/2) - 2]}{R^2(1-Kr^2)^2}$$
$$-e^{-\nu}\{2(\ddot{R}/R) - 2(\ddot{R}/R)^2 - (\dot{R}\dot{\gamma}/R)\} = ((\rho+p)\cosh^2\beta + \sigma(\cosh\beta - \sinh\beta)^2 \tag{40}$$

$$8\pi e^{(\gamma-\alpha)/2}[(\rho+p)\cosh\beta\sinh\beta - \sigma(\cosh\beta - \sinh\beta)] = \frac{(1-kr^2)\gamma'}{R^2(1-Kr^2)}(\ddot{R}/R) \tag{41}$$

EFEs is a system of four equations for determining the four physical parameters $\rho,\ p,\ \sigma$ and β in terms of metric variables $R(t),\ \gamma(r,t)$. The geometrical parameters k and K determine the geometry of the physical 3-space giving rise to

different classes of models. The function of $\xi(r,t)$ depends on the anisotropic nature of stresses.
It follows that

$$8\pi\sigma = \frac{(G^2_2 - G^1_1)(G^2_2 - G^4_4)}{(G^1_1 + G^4_4 - 2G^2_2)}. \tag{42}$$

Subsequently Equations (3.16) and (3.17) determine parameters σ and β.

The space-time in the exterior region will be described by the Vaidya (1966) metric for a radiating star in the form

$$ds^2 = (1-2m/S+2S'/u')u'^2dt^2-(1-2m/S+2S'/u')u'^2dr^2-S^2(d\theta^2+\sin^2\theta d\phi^2), \tag{43}$$

where $m(u)$ and $S(r,t)$ are undetermined functions satisfying

$$(1 - 2m/S + S'/u')u' = S'.$$

We stipulate the standard boundary conditions at the boundary $r = a(t)$ which ensure continuity of metric and continuity of pressure stipulating $p(a) = 0$ and $v^1/v^4 = a$.
If we let $S = rR(t)$ the continuity of metric coefficients g_{22} and g_{33} is ensured and the metric in the exterior region has explicit form

$$ds^2 = (1 - 2m/rR + 2R'r/u')(u'^2dt^2 - u'^2dr^2) - r^2R^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{44}$$

The explicit solutions for different permissible K in the three classes of geometries corresponding to $k = 1, 0, 1$, have been considered. The restriction $0 < K < 1$, in the Vaidya-Patel (1996) procedure obtained in view of their choice $\gamma = 0$ in (25) does not arise in this approach and the analysis holds for all K. The procedure can be used to investigate the nature of the collapse of a spherical stellar configuration comprising of anisotropic matter or perfect fluid matter with charge when its equilibrium is lost.

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