J. T. S. ISSN : 0974-5428
Vol. 4 (2010), pp.9-19

# On Weakly Symmetric and Weakly Ricci-Symmetric Almost $r$-Para Contact Manifolds of LP-Sasakian and Kenmotsu Type

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(Received: November 3, 2009)

**Abstract**

The present paper deals with weakly symmetric and weakly Ricci-symmetric almost $r$-para contact manifolds of LP-Sasakian type and Kenmotsu type. We obtain necessary conditions in order that an almost $r$-para contact manifolds of LP-Sasakian and of Kenmotsu type be weakly symmetric and weakly Ricci-symmetric, respectively .

**Keywords and Phrases :** Almost $r$-para contact manifold , weakly symmetric manifold, weakly Ricci-symmetric manifold.
**2000 AMS Subject Classification :** 53C21, 53C25.

## 1. Introduction

The notions of weakly symmetric and weakly Ricci-symmetric Riemannian manifolds were introduced by L. Tamassy and T. Q. Binh in 1992 and 1993 (see [9], [8]). In 2000, U. C. De, T. Q. Binh and A. A. Shaikh gave necessary conditions for the compatibility of several $k$-contact structures with weak symmetry and weak Ricci-symmetry [4]. In 2002, C. Özgür studied on weak symmetries of Lorentzian para-Sasakian manifolds [10] and also the author considered weakly symmetric Kenmotsu manifolds in [11]. Then N. Aktan and A. Görgülü studied in 2007 on weak symmetries of almost $r$-para contact Riemannian manifold of P-Sasakian type [1]. Here we study weakly symmetric and weakly Ricci-symmetric almost $r$-para contact manifolds of LP-Sasakian type and Kenmotsu type.

## 2. Preliminaries

A non-flat differentiable manifold $(M^n, g)$ $(n > 2)$ is called weakly symmetric if there exist 1-forms $\alpha, \beta, \gamma, \delta$ and $\sigma$ on $M$ such that

$$
\begin{aligned}
(\nabla_X \acute{R})(Y,Z,U,V) &= \alpha(X)\acute{R}(Y,Z,U,V) + \beta(Y)\acute{R}(X,Z,U,V) \\
&\quad + \gamma(Z)\acute{R}(Y,X,U,V) + \delta(U)\acute{R}(Y,Z,X,V) \\
&\quad + \sigma(V)\acute{R}(Y,Z,U,X) \qquad (2.1)
\end{aligned}
$$

holds for vector fields $X, Y, Z, U, V$ on $M$;
where $\acute{R}(X,Y,Z,U) = g(R(X,Y)Z,U)$.

A differentiable manifold $(M^n, g)\,(n > 2)$ is called weakly Ricci symmetric if there exist 1-forms $\rho, \mu, \nu$ such that

$$
(\nabla_X S)(Y,Z) = \rho(X)S(Y,Z) + \mu(Y)S(X,Z) + \nu(Z)S(X,Y) \qquad (2.2)
$$

holds for all vector fields $X, Y, Z$; where $S(X,Y) = g(QX,Y)$,
$Q$ be the symmetric endomorphism of the tangent space of $M$.

If $M$ is weakly symmetric, then from (2.1), we obtain (see [8], [9])

$$
\begin{aligned}
(\nabla_X S)(Z,U) &= \alpha(X)S(Z,U) + \beta(Z)S(X,U) + \delta(U)S(Z,X) \\
&\quad + \beta(R(X,Z)U) + \delta(R(X,U)Z) \qquad (2.3)
\end{aligned}
$$

An $n$-dimensional differentiable manifold $M$ is called a Lorentzian Para-Sasakian (briefly LP-Sasakian) manifold ([6], [7]) if it admits a (1,1) tensor field $\phi$, a contravariant vector field $\xi$, a covariant vector field $\eta$ and a Lorentzian metric $g$ which satisfy

$$\eta(\xi) = -1, \qquad (2.4)$$

$$\phi^2 = I + \eta(X)\xi, \qquad (2.5)$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \qquad (2.6)$$

$$g(X,\xi) = \eta(X), \nabla_X \xi = \phi X, \qquad (2.7)$$

$$(\nabla_X \phi)(Y) = [g(X,Y) + \eta(X)\eta(Y)]\xi + [X + \eta(X)\xi]\eta(Y), \qquad (2.8)$$

where $\nabla$ denotes the operator of covariant differentiation with respect to the Lorentzian metric $g$.

In a LP-Sasakian manifold , the following relations hold

$$\phi\xi = 0, \eta(\phi X) = 0 \qquad (2.9)$$

$$\text{rank}\phi = n - 1. \qquad (2.10)$$

Let $(M, \phi, \xi, \eta, g)$ be an $n$-dimensional almost contact Riemannian manifold, where $\phi$ is a (1,1) tensor field, $\xi$ is the structure vector field, $\eta$ is a 1-form and $g$ is a Riemannian metric. It is well known $(\phi, \xi, \eta, g)$ satisfy the following [2]:

$$\eta(\xi) = 1, \tag{2.11}$$

$$g(X, \xi) = \eta(X), \tag{2.12}$$

$$\phi^2 X = -X + \eta(X)\xi, \tag{2.13}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.14}$$

$$\phi(\xi) = 0, \tag{2.15}$$

$$\eta(\phi X) = 0, \tag{2.16}$$

$\forall$ vector fields $X, Y$ on $M$.

If moreover,

$$(\nabla_X \phi)Y = -g(X, \phi Y)\xi - \eta(Y)\phi(X), \tag{2.17}$$

where $\nabla$ denotes the Riemannian connection, then $(M, \phi, \xi, \eta, g)$ is called a Kenmotsu manifold [5]. In a Kenmotsu manifold, the following property holds

$$\nabla_X \xi = X - \eta(X)\xi. \tag{2.18}$$

A differentiable manifold $(M, g)$ of dimension $(n + r)$ with tangent space $T(M)$ is said to be an almost $r$-para contact Riemannian manifold (by [3]) if there exist a tensor field $\phi$ of type $(1, 1)$ and $r$ global vector fields $\xi_1, \dots, \xi_r$ (called structure vector fields) such that

i) if $\eta_1, \dots, \eta_r$ are dual 1-forms of $\xi_1, \dots \xi_r$; then

$$\eta_i(\xi_j) = \delta^i_j;$$

$$g(\xi_i, X) = \eta_i(X);$$

$$\phi^2 = I - \sum_{i=1}^{r} \xi_i \otimes \eta_i \tag{2.19}$$

ii) $$g(\phi\, X, \phi\, Y) = g(X, Y) - \sum_{i=1}^{r} \eta_i(X)\eta_i(Y), \tag{2.20}$$

for $X, Y \in T(M)$.

We define an almost $r$-para contact manifold of LP-Sasakian type as follows:

**Definition (2.1) :** An almost $r$-para contact manifold $M$ is said to be of LP-Sasakian type if

$$\nabla_X \xi_i = \phi\, X \tag{2.21}$$

$$(\nabla_X \phi)(Y) = \sum_{i=1}^{r}[g(X,Y) + \eta_i(X)\eta_i(Y)]\xi_i + \sum_{i=1}^{r}[X + \eta_i(X)\xi_i]\eta_i(Y), \tag{2.22}$$

$\forall X, Y \in T(M)$.

In an almost $r$-para contact manifold of LP-Sasakian type $M$, the following relations hold

$$S(\xi_i, X) = (n-1)\ \sum_{i=1}^{r} \eta_i(X) \tag{2.23}$$

$$R(\xi_i, X)\xi_i = X + \sum_{i=1}^{r} \eta_i(X)\xi_i \tag{2.24}$$

$$g(R(\xi_i, X)Y, \xi_i) = \sum_{i=1}^{r}[g(X,Y)\eta_i(\xi_i) - g(\xi_i, Y)\eta_i(X)] \tag{2.25}$$

for vector fields $X, Y \in T(M)$.

Again we define an almost $r$-para contact Riemannian manifold of Kenmotsu type as follows:

**Definition (2.2) :** An almost $r$-para contact Riemannian manifold $M$ is said to be of Kenmotsu type if

$$\nabla_X \xi_i = X - \sum_{i=1}^{r} \eta_i(X)\xi_i \tag{2.26}$$

$$(\nabla_X \phi)(Y) = \sum_{i=1}^{r}[-g(X, \phi\, Y)\xi_i - \eta_i(Y)\phi(X)], \tag{2.27}$$

$\forall X, Y \in\ T(M)$.

In an almost $r$-para contact Riemannian manifold of Kenmotsu type $M$, the following relations hold

$$S(\xi_i, X) = -(n-1)\ \sum_{i=1}^{r} \eta_i(X) \tag{2.28}$$

$$R(\xi_i, X)\xi_i = X - \sum_{i=1}^{r} \eta_i(X)\xi_i \tag{2.29}$$

$$g(R(\xi_i, X)Y, \xi_i) = -g(X, Y) + \sum_{i=1}^{r} \eta_i(X)\eta_i(Y) \tag{2.30}$$

for vector fields $X, Y \in T(M)$.

Since $\phi$ is skew symmetric and the Ricci operator $Q$ is symmetric in an almost $r$-para contact manifold of LP-Sasakian type (or Kenmotsu type), $Q\,\phi + \phi\,Q = 0$ and thus the Lie derivative of $S$ vanishes i.e.,

$$L_{\xi_i} S = 0. \tag{2.31}$$

for any $i = 1, \ldots, r$.

## 3. Weakly symmetric almost $r$-para contact manifold of LP-Sasakian type

In this section we suppose that the considered weakly symmetric manifold is almost $r$-para contact manifold of LP-Sasakian type. Then we obtain

**Theorem 3.1 :** Any weakly symmetric almost $r$-para contact manifold of LP-Sasakian type $M$, satisfies $\alpha + \beta + \delta = 0$.

**Proof :** Since the manifold is weakly symmetric, by putting $X = \xi_i$ in (2.3), we have

$$\begin{aligned}(\nabla_{\xi_i} S)(Z, U) = \alpha(\xi_i)S(Z, U) + \beta(Z)S(\xi_i, U) + \delta(U)S(Z, \xi_i) \\ +\beta(R(\xi_i, Z)U) + \delta(R(\xi_i, U)Z)\end{aligned} \tag{3.1}$$

By virtue of (2.21) and (2.31) we obtain

$$(\nabla_{\xi_i} S)(Z, U) = 0 \tag{3.2}$$

From (3.1) and (3.2), we have

$$\begin{aligned}\alpha(\xi_i)S(Z, U) + \beta(Z)S(\xi_i, U) + \delta(U)S(Z, \xi_i) \\ +\beta(R(\xi_i, Z)U) + \delta(R(\xi_i, U)Z) = 0\end{aligned} \tag{3.3}$$

Putting $Z = U = \xi_i$ in (3.3) and using (2.24), we get

$$[\alpha(\xi_i) + \beta(\xi_i) + \delta(\xi_i)]S(\xi_i, \xi_i) = 0 \tag{3.4}$$

which gives

$$\alpha(\xi_i) + \beta(\xi_i) + \delta(\xi_i) = 0. \tag{3.5}$$

This shows that $\alpha + \beta + \delta = 0$ over the vector field $\xi_i$ on $M$.

Now we will show that $\alpha + \beta + \delta = 0$ holds for all vector fields on $M$.

Taking $X = Z = \xi_i$ in (2.3), we obtain

$$(\nabla_{\xi_i} S)(\xi_i, U) = \alpha(\xi_i)S(\xi_i, U) + \beta(\xi_i)S(\xi_i, U) + \delta(U)S(\xi_i, \xi_i) \\ +\beta(R(\xi_i, \xi_i)U) + \delta(R(\xi_i, U)\xi_i) \quad (3.6)$$

Replacing $U$ by $X$ in (3.6), we get

$$\alpha(\xi_i)S(\xi_i, X) + \beta(\xi_i)S(\xi_i, X) + \delta(X)S(\xi_i, \xi_i) \\ +\beta(R(\xi_i, \xi_i)X) + \delta(R(\xi_i, X)\xi_i) = 0 \quad (3.7)$$

In (2.3), taking $X = U = \xi_i$, we have

$$(\nabla_{\xi_i} S)(Z, \xi_i) = \alpha(\xi_i)S(Z, \xi_i) + \beta(Z)S(\xi_i, \xi_i) + \delta(\xi_i)S(Z, \xi_i) \\ +\beta(R(\xi_i, Z)\xi_i) + \delta(R(\xi_i, \xi_i)Z) \quad (3.8)$$

Using (3.2) in (3.8) and replacing $Z$ by $X$, we obtain

$$\alpha(\xi_i)S(X, \xi_i) + \beta(X)S(\xi_i, \xi_i) + \delta(\xi_i)S(X, \xi_i) \\ +\beta(R(\xi_i, X)\xi_i) + \delta(R(\xi_i, \xi_i)X) = 0 \quad (3.9)$$

In (2.3), taking $Z = U = \xi_i$, we have

$$(\nabla_X S)(\xi_i, \xi_i) = \alpha(X)S(\xi_i, \xi_i) + \beta(\xi_i)S(X, \xi_i) + \delta(\xi_i)S(\xi_i, X) \\ +\beta(R(X, \xi_i)\xi_i) + \delta(R(X, \xi_i)\xi_i) \quad (3.10)$$

Here also we have

$$(\nabla_X S)(\xi_i, \xi_i) = 0 \quad (3.11)$$

Using (3.11) in (3.10), we obtain

$$\alpha(X)S(\xi_i, \xi_i) + \beta(\xi_i)S(X, \xi_i) + \delta(\xi_i)S(\xi_i, X) \\ +\beta(R(X, \xi_i)\xi_i) + \delta(R(X, \xi_i)\xi_i) = 0 \quad (3.12)$$

adding (3.7), (3.9) and (3.12) and then using (3.5), we get

$$[\alpha(X) + \beta(X) + \delta(X)]S(\xi_i, \xi_i) = 0 \quad (3.13)$$

Hence from (3.13), we obtain

$$\alpha(X) + \beta(X) + \delta(X) = 0, \qquad \forall X.$$

Thus

$$\alpha + \beta + \delta = 0.$$

Hence the theorem is proved.

## 4. Weakly Ricci-symmetric almost $r$-para contact manifold of LP-Sasakian type

In this section we suppose that the weakly Ricci-symmetric manifold is almost $r$-para contact manifold of LP-Sasakian type. Then we have

**Theorem 4.1 :** Any weakly Ricci-symmetric almost $r$-para contact manifold of LP-Sasakian type $M$ satisfies $\rho + \mu + \nu = 0$.

**Proof.** Since $M$ is weakly Ricci-symmetric almost $r$-para contact manifold of LP-Sasakian type, then

by putting $X = \xi_i$ in (2.2) we get

$$(\nabla_{\xi_i} S)(Y, Z) = \rho(\xi_i)S(Y, Z) + \mu(Y)S(\xi_i, Z) + \nu(Z)S(\xi_i, Y) \tag{4.1}$$

Using (3.2) in (4.1), we have

$$\rho(\xi_i)S(Y, Z) + \mu(Y)S(\xi_i, Z) + \nu(Z)S(\xi_i, Y) = 0 \tag{4.2}$$

Replacing $Y$ and $Z$ by $\xi_i$ in (4.2), we obtain

$$[\rho(\xi_i) + \mu(\xi_i) + \nu(\xi_i)]S(\xi_i, \xi_i) = 0 \tag{4.3}$$

which gives

$$\rho(\xi_i) + \mu(\xi_i) + \nu(\xi_i) = 0 \tag{4.4}$$

Taking $X = Y = \xi_i$ in (2.2) and using (3.2), then putting $Z = X$, we get

$$\rho(\xi_i)S(\xi_i, X) + \mu(\xi_i)S(\xi_i, X) + \nu(X)S(\xi_i, \xi_i) = 0. \tag{4.5}$$

In (2.2), taking $X = Z = \xi_i$ and using (3.2), then replacing $Y$ by $X$, we obtain

$$\rho(\xi_i)S(X, \xi_i) + \mu(X)S(\xi_i, \xi_i) + \nu(\xi_i)S(\xi_i, X) = 0 \tag{4.6}$$

Putting $Y = Z = \xi_i$ in (2.2) and using (3.11), we obtain

$$\rho(X)S(\xi_i, \xi_i) + \mu(\xi_i)S(X, \xi_i) + \nu(\xi_i)S(X, \xi_i) = 0 \tag{4.7}$$

Adding (4.5), (4.6) and (4.7) and then using (4.4), we have

$$[\rho(X) + \mu(X) + \nu(X)]S(\xi_i, \xi_i) = 0 \tag{4.8}$$

Now from (4.8), we have

$$\rho(X) + \mu(X) + \nu(X) = 0, \qquad \forall X.$$

Thus

$$\rho + \mu + \nu = 0.$$

Hence the theorem is proved.

## 5. Weakly symmetric almost $r$-para contact Riemannian manifold of Kenmotsu type

Here we assume that the weakly symmetric manifold is almost $r$-para contact Riemannian manifold of Kenmotsu type. Then we have

**Theorem 5.1 :** Any weakly symmetric almost $r$-para contact Riemannian manifold of Kenmotsu type $M$ satisfies $\alpha + \beta + \delta = 0$.

**Proof .** Since $M$ is weakly symmetric, by taking $X = \xi_i$ in (2.3), we have

$$(\nabla_{\xi_i} S)(Z, U) = \alpha(\xi_i)S(Z, U) + \beta(Z)S(\xi_i, U) + \delta(U)S(Z, \xi_i)$$
$$+\beta(R(\xi_i, Z)U) + \delta(R(\xi_i, U)Z) \qquad (5.1)$$

By virtue of (2.26) and (2.31), we obtain

$$(\nabla_{\xi_i} S)(Z, U) = 0 \qquad (5.2)$$

From (5.1) and (5.2), we have

$$\alpha(\xi_i)S(Z, U) + \beta(Z)S(\xi_i, U) + \delta(U)S(Z, \xi_i)$$
$$+\beta(R(\xi_i, Z)U) + \delta(R(\xi_i, U)Z) = 0 \qquad (5.3)$$

Putting $Z = U = \xi_i$ in (5.3) and using (2.29), we get

$$[\alpha(\xi_i) + \beta(\xi_i) + \delta(\xi_i)]S(\xi_i, \xi_i) = 0 \qquad (5.4)$$

which gives

$$\alpha(\xi_i) + \beta(\xi_i) + \delta(\xi_i) = 0. \qquad (5.5)$$

This shows that $\alpha + \beta + \delta$ vanishes over the vector field $\xi_i$ on $M$.

Now we will show that $\alpha + \beta + \delta = 0$ holds for all vector fields on $M$. In (2.3), taking $X = Z = \xi_i$, we obtain

$$(\nabla_{\xi_i} S)(\xi_i, U) = \alpha(\xi_i)S(\xi_i, U) + \beta(\xi_i)S(\xi_i, U) + \delta(U)S(\xi_i, \xi_i)$$
$$+\beta(R(\xi_i, \xi_i)U) + \delta(R(\xi_i, U)\xi_i) \qquad (5.6)$$

By putting $U = X$ in (5.6), we get

$$\alpha(\xi_i)S(\xi_i, X) + \beta(\xi_i)S(\xi_i, X) + \delta(X)S(\xi_i, \xi_i)$$
$$+\beta(R(\xi_i, \xi_i)X) + \delta(R(\xi_i, X)\xi_i) = 0 \qquad (5.7)$$

In (2.3), taking $X = U = \xi_i$, we get

$$(\nabla_{\xi_i} S)(Z, \xi_i) = \alpha(\xi_i)S(Z, \xi_i) + \beta(Z)S(\xi_i, \xi_i) + \delta(\xi_i)S(Z, \xi_i) + \beta(R(\xi_i, Z)\xi_i) + \delta(R(\xi_i, \xi_i)Z) \tag{5.8}$$

Using (5.2) in (5.8) and then replacing $Z$ by $X$, we have

$$\alpha(\xi_i)S(X, \xi_i) + \beta(X)S(\xi_i, \xi_i) + \delta(\xi_i)S(X, \xi_i) + \beta(R(\xi_i, X)\xi_i) + \delta(R(\xi_i, \xi_i)X) = 0 \tag{5.9}$$

Again in (2.3), taking $Z = U = \xi_i$, we get

$$(\nabla_X S)(\xi_i, \xi_i) = \alpha(X)S(\xi_i, \xi_i) + \beta(\xi_i)S(X, \xi_i) + \delta(\xi_i)S(\xi_i, X) + \beta(R(X, \xi_i)\xi_i) + \delta(R(X, \xi_i)\xi_i) \tag{5.10}$$

Here also we have

$$(\nabla_X S)(\xi_i, \xi_i) = 0 \tag{5.11}$$

Using (5.11) in (5.10), we obtain

$$\alpha(X)S(\xi_i, \xi_i) + \beta(\xi_i)S(X, \xi_i) + \delta(\xi_i)S(\xi_i, X) + \beta(R(X, \xi_i)\xi_i) + \delta(R(X, \xi_i)\xi_i) = 0 \tag{5.12}$$

adding (5.7), (5.9) and (5.12) and then using (5.5), we get

$$[\alpha(X) + \beta(X) + \delta(X)]S(\xi_i, \xi_i) = 0 \tag{5.13}$$

Hence from (5.13), we obtain

$$\alpha(X) + \beta(X) + \delta(X) = 0, \qquad \forall X.$$

Thus

$$\alpha + \beta + \delta = 0.$$

Hence the theorem is proved.

## 6. Weakly Ricci-symmetric almost $r$-para contact Riemannian manifold of Kenmotsu type

We suppose that the weakly Ricci-symmetric manifold is almost $r$-para contact Riemannian manifold of Kenmotsu type. Then we have

**Theorem 6.1 :** Any weakly Ricci-symmetric almost $r$-para contact Riemannian manifold of Kenmotsu type $M$ satisfies $\rho + \mu + \nu = 0$.

**Proof .** Since $M$ is weakly Ricci-symmetric almost $r$-para contact Riemannian manifold of Kenmotsu type,

Putting $X = \xi_i$ in (2.2) we get

$$(\nabla_{\xi_i} S)(Y, Z) = \rho(\xi_i)S(Y, Z) + \mu(Y)S(\xi_i, Z) + \nu(Z)S(\xi_i, Y) \tag{6.1}$$

Using (5.2) in (6.1), we have

$$\rho(\xi_i)S(Y, Z) + \mu(Y)S(\xi_i, Z) + \nu(Z)S(\xi_i, Y) = 0 \tag{6.2}$$

Replacing $Y$ and $Z$ by $\xi_i$ in (6.2), we obtain

$$[\rho(\xi_i) + \mu(\xi_i) + \nu(\xi_i)]S(\xi_i, \xi_i) = 0 \tag{6.3}$$

which gives

$$\rho(\xi_i) + \mu(\xi_i) + \nu(\xi_i) = 0 \tag{6.4}$$

Taking $X = Y = \xi_i$ in (2.2) and using (5.2), then replacing $Z$ by $X$, we obtain

$$\rho(\xi_i)S(\xi_i, X) + \mu(\xi_i)S(\xi_i, X) + \nu(X)S(\xi_i, \xi_i) = 0 \tag{6.5}$$

In (2.2), taking $X = Z = \xi_i$ and using (5.2), we get

$$\rho(\xi_i)S(Y, \xi_i) + \mu(Y)S(\xi_i, \xi_i) + \nu(\xi_i)S(\xi_i, Y) = 0 \tag{6.6}$$

Replacing $Y$ by $X$ in (6.6), we have

$$\rho(\xi_i)S(X, \xi_i) + \mu(X)S(\xi_i, \xi_i) + \nu(\xi_i)S(\xi_i, X) = 0 \tag{6.7}$$

Putting $Y = Z = \xi_i$ in (2.2) and using (5.11), we obtain

$$\rho(X)S(\xi_i, \xi_i) + \mu(\xi_i)S(X, \xi_i) + \nu(\xi_i)S(X, \xi_i) = 0 \tag{6.8}$$

Adding (6.5), (6.7) and (6.8) and then using (6.4), we have

$$[\rho(X) + \mu(X) + \nu(X)]S(\xi_i, \xi_i) = 0 \tag{6.9}$$

Now from (6.9), we have

$$\rho(X) + \mu(X) + \nu(X) = 0, \qquad \forall X.$$

Thus

$$\rho + \mu + \nu = 0.$$

Hence the theorem is proved.

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