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On Weak Concircular Symmetries of Para-Sasakian Manifold admitting Quarter-symmetric Metric Connection

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Abstract

The object of the present paper is to study weakly concircular symmetric, weakly concircular Ricci-symmetric and special weakly concircular Ricci-symmetric para-Sasakian manifold with respect to quarter-symmetric metric connection.

Key Words : weakly concircular symmetric, weakly concircular Ricci-symmetric, special weakly concircular Ricci-symmetric, para-Sasakian manifold, quarter-symmetric metric connection.
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1. Introduction

In 1950 Norwegian mathematician Atle Selberg introduced the concept of weakly symmetric space. Geometrically, these spaces are defined as complete Riemannian manifolds such that any two points can be exchanged by an isometry, the symmetric case being when the isometry is required to have period two. The classification of weakly symmetric spaces relies on that of periodic automorphism of complex bi-semi simple Lie algebras.

As a proper generalization of Chaki's [3, 4] Pseudo symmetric and Pseudo Ricci-symmetric manifolds, the concept of weakly symmetric and weakly projective symmetric manifolds were introduced by Tamassy and Binh [25, 26]. Later Binh [26] studied decomposable weakly symmetric manifolds. A non-flat Riemannian manifold (M^n, g) $(n > 2)$ is called weakly symmetric if its curvature tensor R of type $(0, 4)$ satisfies the condition

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$$(\nabla_W R)(Y,Z,U,V) = D(X)R(Y,Z,U,V) + E(Y)R(X,Z,U,V) + G(Z)R(Y,X,U,V) + F(U)R(Y,Z,X,V) + H(V)R(Y,Z,U,X), \tag{1.1}$$

for all vector fields $X, Y, Z, U, V \in \chi(M^n)$, $\chi(M^n)$ being the Lie Algebra of the smooth vector fields of M^n, where D, E, F, G, and H are 1-forms (not simultaneously zero) and ∇ denotes the operator of covariant differentiation with respect to the Riemannian metric g. The 1-forms are called the associated 1-forms of the manifold and an n-dimensional manifold of this kind is denoted by $(WS)_n$. If in (1.1), the 1-form D is replaced by $2D$ and H is replaced by D, then a $(WS)_n$ reduces to the notion of generalized pseudo symmetric manifold by Chaki [3]. In 1999, the authors [5] studied $(WS)_n$ and proved that in such a manifold the associated 1-forms $E = G$ and $F = H$. Hence (1.1) reduces to the following form:

$$(\nabla_W R)(Y,Z,U,V) = D(X)R(Y,Z,U,V) + E(Y)R(X,Z,U,V) + E(Z)R(Y,X,U,V) + F(U)R(Y,Z,X,V) + F(V)R(Y,Z,U,X). \tag{1.2}$$

Later Tamassy and Binh [26] introduced the concept of weakly Ricci symmetric manifolds. A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly Ricci symmetric manifold if its Ricci tensor S of type $(0, 2)$ is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = D(X)S(Y,Z) + E(Y)S(X,Z) + F(Z)S(Y,X), \tag{1.3}$$

where D, E and F are non-zero 1-forms and these are called associated 1-forms of the manifold. If $D = E = F$ then the manifold is called pseudo Ricci symmetric.

Many authors have studied weak symmetries of Sasakian manifold, Kenmotsu manifold and trans-Sasakian manifold in [26],[14],[22] and others.

In [24], Singh and Quddus Khan introduced and studied the notion of special weakly Ricci symmetric manifolds. Special weakly Ricci symmetric Kenmotsu manifolds were studied by Nesip Aktan et al. [13].

A Riemannian manifold $(M^n, g)(n > 2)$ is called a special weakly Ricci symmetric if

$$(\nabla_X S)(Y,Z) = 2\alpha(X)S(Y,Z) + \alpha(Y)S(X,Z) + \alpha(Z)S(Y,X), \tag{1.4}$$

where α is a 1-form and is defined as

$$\alpha(X) = g(X, \rho),$$

where ρ is an associated vector field.

A transformation of n-dimensional Riemannian manifold M^n, which transforms every geodesic circle of M^n into a geodesic circle, is called a concircular transformation [27] and is defined by

$$\bar{C}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y], \tag{1.5}$$

where $\bar{C}$ and r are the concircular curvature tensor and scalar curvature of the manifold respectively. Let $\{e_1, e_2, e_3, ..., e_n\}$ be an orthonormal basis of the tangent space at each point of the manifold and let

$$\bar{P}(Y, V) = \sum_{i=1}^{n} \bar{C}(Y, e_i, e_i, Z) = S(Y, V) - \frac{r}{n} g(Y, V), \tag{1.6}$$

where $\bar{C}$ and $\bar{P}$ are the concircular curvature tensor and concircular Ricci-symmetric tensor respectively.

Recently, the geometers [22, 7] introduced the concept of weakly concircular symmetric manifold and weakly concircular Ricci symmetric manifold respectively. Further weak concircular symmetries on different manifolds were studied in [21, 7, 23] and others.

In 1924, the idea of a semi-symmetric linear connection on a differentiable manifold was introduced by Friedmann and Schouten [8]. In 1930, E. Bartolotti [2] gave a geometrical meaning of such a connection. Further, the idea of metric connection with torsion on a Riemannian manifold was introduced by Hayden [10] and later Yano [29] studied some curvature conditions with semi-symmetric connections on Riemannian manifolds. Golab [9] defined and studied a quarter-symmetric connection in a differentiable manifold.

A linear connection $\tilde{\nabla}$ on an n-dimensional differentiable manifold is said to be a quarter-symmetric connection [9] if its torsion tensor T is of the form

$$T(X, Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X, Y] = \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.7}$$

where η is a 1-form and ϕ is a tensor of type $(1, 1)$. In particular, if we replace ϕX by X and ϕY by Y, then the quarter-symmetric connection reduces to the semisymmetric connection [8]. Thus, the notion of quarter-symmetric connection generalizes the idea of semisymmetric connection. And if quarter-symmetric linear connection $\tilde{\nabla}$ satisfies the condition

$$(\tilde{\nabla}_X g)(Y, Z) = 0, \tag{1.8}$$

for all X, Y, $Z \in \chi(M^n)$, where $\chi(M^n)$ is the Lie algebra of vector fields on the manifold M^n, then $\tilde{\nabla}$ is said to be a quarter-symmetric metric connection.

Various properties of quarter-symmetric metric connection have been studied by many authors like Rastogi [17, 18], Mishra and Pandey [12], Yano and Imai [30], De et al. [6], Pradeep Kumar et al. [15, 16, 28] and others.

Motivated by the above ideas, in this paper we made an attempt to study weak concircular symmetries admitting quarter-symmetric metric connection in a para-Sasakian manifold. The present paper is organized as follows: Section 2 is devoted to the preliminary results of para-Sasakian manifold that are needed in the rest of sections. In the following sections 3, 4 and 5, we study weakly concircular symmetric, weakly concircular Ricci-symmetric and special weakly concircular Ricci-symmetric para-Sasakian manifold admitting quarter-symmetric metric connection.

2. Preliminaries

An n-dimensional differentiable manifold M^n is called an almost paracontact manifold if it admits an almost paracontact structure (ϕ, ξ, η) consisting of a $(1, 1)$ tensor field ϕ, a vector field ξ, and a 1-form η satisfying

$$\phi^2 X = X - \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = 1, \;\; \phi \cdot \xi = 0, \;\; \eta \cdot \phi = 0. \tag{2.2}$$

If g is a compatible Riemannian metric with (ϕ, ξ, η), that is,

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \;\; g(X, \xi) = \eta(X), \tag{2.3}$$

$$g(X, \phi Y) = g(\phi X, Y), \tag{2.4}$$

for all vector fields X and Y on M^n, then M^n becomes an almost paracontact Riemannian manifold equipped with an almost paracontact Riemannian structure (ϕ, ξ, η, g). If (ϕ, ξ, η, g) satisfy the following equations:

$$\nabla_X \xi = \phi X, \;\; (\nabla_X \phi)(Y) = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.5}$$

then M^n is called a para-Sasakian manifold or briely a P-Sasakian manifold [1]. Especially, a P-Sasakian manifold M^n is called a special para-Sasakian manifold or briely a SP-Sasakian manifold [20] if M^nadmits a 1-form η satisfying

$$(\nabla_X \eta)(Y) = -g(X,Y) + \eta(X)\eta(Y). \tag{2.6}$$

In an n-dimensional para-Sasakian manifold M^n, the following relations hold [1, 19]:

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{2.7}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.8}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{2.9}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.10}$$

for any vector fields X, Y and Z, where R and S are the Riemannian curvature tensor and the Ricci tensor of M^n respectively.

Now, a quarter-symmetric metric connection $\tilde{\nabla}$ in a para-Sasakian manifold is given by [16]

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g(\phi X, Y)\xi. \tag{2.11}$$

A relation between the curvature tensor of M^n with respect to the quarter-symmetric metric connection $\tilde{\nabla}$ and the Riemannian connection ∇ is given by [16, 11]

$$\begin{aligned}\tilde{R}(X,Y)Z =& R(X,Y)Z + 3g(\phi X, Z)\phi Y - 3g(\phi Y, Z)\phi X + \eta(Z)[\eta(X)Y \\ & - \eta(Y)X] - [\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi,\end{aligned} \tag{2.12}$$

where $\tilde{R}$ and R denote the Riemannian curvature of the connection $\tilde{\nabla}$ and ∇ respectively. From (2.12), it follows that

$$\tilde{S}(Y,Z) = S(Y,Z) + 2g(Y,Z) - (n+1)\eta(Y)\eta(Z) - 3trace\phi g(\phi Y, Z), \tag{2.13}$$

where $\tilde{S}$ and S are Ricci curvatures of connections $\tilde{\nabla}$ and ∇ respectively.

3. Weakly concircular symmetric Para-Sasakian manifold admitting quarter-symmetric metric connection

A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly concircular symmetric manifold $(W\bar{C}S)_n$ [22] if its concircular curvature tensor $\bar{C}$ satisfies the following condition

$$\begin{aligned}(\nabla_W \bar{C})(Y,Z,U,V) =& D(X)\bar{C}(Y,Z,U,V) + E(Y)\bar{C}(X,Z,U,V) \\ & + E(Z)\bar{C}(Y,X,U,V) + F(U)\bar{C}(Y,Z,X,V) \\ & + F(V)\bar{C}(Y,Z,U,X),\end{aligned} \tag{3.1}$$

for all vector fields X, Y, Z, U, $V \in \chi(M^n)$, where D, E and F are 1-forms (not simultaneously zero) and an n-dimensional manifold of this kind is denoted by $(W\bar{C}S)_n$. In virtue of quarter-symmetric metric connection, (1.6) takes the form

$$\tilde{\bar{P}}(Y,V) = \sum_{i=1}^{n} \tilde{\bar{C}}(Y, e_i, e_i, V) = \tilde{S}(Y,V) - \frac{\tilde{r}}{n} g(Y,V), \tag{3.2}$$

where $\tilde{\bar{P}}$ is a concircular Ricci symmetric tensor. In the sense of quarter-symmetric metric connection, we can define the following definition;

Definition 3.1. A para-Sasakian manifold M^n $(n > 2)$ is called weakly concircular symmetric with respect to quarter-symmetric metric connection $\tilde{\nabla}$ if there exists 1-forms D, E and F and concircular curvature tensor $\bar{C}$ satisfying the following condition

$$\begin{aligned} (\tilde{\nabla}_X \tilde{\bar{C}})(Y,Z,U,V) =& D(X)\tilde{\bar{C}}(Y,Z,U,V) + E(Y)\tilde{\bar{C}}(X,Z,U,V) \\ &+ E(Z)\tilde{\bar{C}}(Y,X,U,V) + F(U)\tilde{\bar{C}}(Y,Z,X,V) \\ &+ F(V)\tilde{\bar{C}}(Y,Z,U,X), \end{aligned} \tag{3.3}$$

for all vector fields X, Y, Z, U, $V \in \chi(M^n)$.

Theorem 3.1. In a weakly concircular symmetric para-Sasakian manifold (M^n, g) $(n > 2)$ admitting quarter-symmetric metric connection $\tilde{\nabla}$, the associated 1-forms D, E and F are given by (3.8), (3.10) and (3.11), respectively.

Proof: Let M^n be a weakly concircular symmetric para-Sasakian manifold with respect to quarter-symmetric metric connection $\tilde{\nabla}$. Therefore (3.3) holds. On contraction and using (3.2), (3.3) reduces to

$$\begin{aligned} (\tilde{\nabla}_X \tilde{S})(Z,U) - \frac{d\tilde{r}(X)}{n} g(Z,U) =& D(X)[\tilde{S}(Z,U) - \frac{\tilde{r}}{n} g(Z,U)] \\ &+ E(Z)[\tilde{S}(X,U) - \frac{\tilde{r}}{n} g(X,U)] + F(U)[\tilde{S}(Z,X) \\ &- \frac{\tilde{r}}{n} g(Z,X)] + E(\tilde{R}(X,Z)U) + F(\tilde{R}(X,U)Z) \\ &- \frac{\tilde{r}}{n(n-1)}[\{E(X) + F(X)\} g(Z,U) \\ &- E(Z) g(X,U) - F(U) g(Z,X)]. \end{aligned} \tag{3.4}$$

Considering $X = Z = U = \xi$ in (3.4) and using (2.2), (2.9) and (2.13), we obtain

$$D(\xi) + E(\xi) + F(\xi) = \frac{d\tilde{r}(\xi)}{\tilde{r} + 2n(n-1)}. \tag{3.5}$$

Continuing the process by plugging $X = Z = \xi$ in (3.4) and in view of (2.3), (2.9) and (2.13), we get

$$\begin{aligned}(\tilde{\nabla}_\xi \tilde{S})(\xi, U) - \frac{d\tilde{r}(\xi)}{n}\eta(U) = &- [D(\xi) + E(\xi)][\frac{\tilde{r} + 2n(n-1)}{n}]\eta(U) \\ &+ [\frac{\tilde{r}}{n(n-1)} - \frac{\tilde{r} + 2n(n-1)}{n}]F(U) \\ &- [\frac{\tilde{r}}{n(n-1)}]F(\xi)\eta(U).\end{aligned} \tag{3.6}$$

Also from (2.13), we have

$$(\tilde{\nabla}_\xi \tilde{S})(\xi, U) = 0. \tag{3.7}$$

On applying (3.5) and (3.7) in (3.6), we get the relation

$$F(U) = F(\xi)\eta(U). \tag{3.8}$$

Again by taking $X = U = \xi$ in (3.4) and in virtue of (2.3), (2.9) and (2.13), then we obtain

$$\begin{aligned}(\tilde{\nabla}_\xi \tilde{S})(Z, \xi) - \frac{d\tilde{r}(\xi)}{n}\eta(Z) = &- [D(\xi) + F(\xi)][\frac{\tilde{r} + 2n(n-1)}{n}]\eta(Z) \\ &+ [\frac{\tilde{r}}{n(n-1)} - \frac{\tilde{r} + 2n(n-1)}{n}]E(Z) \\ &- [\frac{\tilde{r}}{n(n-1)}]E(\xi)\eta(Z).\end{aligned} \tag{3.9}$$

In view of (3.5) and (3.7), (3.9) leads to

$$E(Z) = E(\xi)\eta(Z). \tag{3.10}$$

Similarly, treating $Z = U = \xi$ in (3.4) and proceeding in the similar manner as above, we have

$$D(X) = \frac{d\tilde{r}(X)}{\tilde{r} + 2n(n-1)} - \frac{d\tilde{r}(\xi)}{\tilde{r} + 2n(n-1)}\eta(X) + D(\xi)\eta(X). \tag{3.11}$$

Hence, this completes the proof.

4. Weakly concircular Ricci-symmetric Para-Sasakian manifold admitting quarter-symmetric metric connection

A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly concircular Ricci symmetric manifold [7] if its concircular Ricci tensor P of type $(0, 2)$ is not identically zero and satisfies the condition

$$(\nabla_X P)(Y, Z) = D(X)P(Y, Z) + E(Y)P(X, Z) + F(Z)P(Y, X), \tag{4.1}$$

where D, E and F are three 1-forms (not simultaneously zero). If $D = E = F$ then it is called pseudo concircular Ricci symmetric. With the help of (4.1), we can define the following definition;

Definition 4.1. A para-Sasakian manifold (M^n, g) $(n > 2)$ is called weakly concircular Ricci-symmetric with respect to quarter-symmetric metric connection $\tilde{\nabla}$ if there exists 1-forms D, E and F and the concircular curvature tensor $\bar{P}$ satisfies the following condition

$$(\tilde{\nabla}_X \tilde{\bar{P}})(Y, Z) = D(X)\tilde{\bar{P}}(Y, Z) + E(Y)\tilde{\bar{P}}(X, Z) + F(Z)\tilde{\bar{P}}(Y, X), \tag{4.2}$$

for all vector fields X, Y, $Z \in \chi(M^n)$.

Theorem 4.1. In a weakly concircular Ricci-symmetric para-Sasakian manifold (M^n, g) $(n > 2)$ admitting quarter-symmetric metric connection, the sum of the associated 1-forms vanishes if the scalar curvature $\tilde{r}$ is constant and $\{\tilde{r} + 2n(n-1)\} \neq 0$.

Proof: With the help of (3.2), (4.2) leads to

$$\begin{aligned}(\tilde{\nabla}_X \tilde{S})(Y, Z) - \frac{d\tilde{r}(X)}{n} g(Y, Z) = D(X)[\tilde{S}(Y, Z) - \frac{\tilde{r}}{n} g(Y, Z)] + E(Y)\\ [\tilde{S}(X, Z) - \frac{\tilde{r}}{n} g(X, Z)] + F(Z)[\tilde{S}(Y, X) - \frac{\tilde{r}}{n} g(Y, X)].\end{aligned} \tag{4.3}$$

Setting $X = Y = Z = \xi$ in (4.3) and by virtue of (2.2), (2.9) and (2.13), we obtain

$$D(\xi) + E(\xi) + F(\xi) = \frac{d\tilde{r}(\xi)}{\tilde{r} + 2n(n-1)}. \tag{4.4}$$

Now treating X and Y by ξ in (4.3) and using (2.3), (2.9) and (2.13), we get

$$\begin{aligned}(\tilde{\nabla}_\xi \tilde{S})(\xi, Z) - [\frac{d\tilde{r}(X)}{n}]\eta(Z) =& [D(\xi) + E(\xi)]\{\frac{\tilde{r} + 2n(n-1)}{n}\}\\ &- F(Z)[\frac{\tilde{r} + 2n(n-1)}{n}].\end{aligned} \tag{4.5}$$

Using (2.13), we obtain

$$(\tilde{\nabla}_\xi \tilde{S})(\xi, Z) = 0. \tag{4.6}$$

Applying (4.4) and (4.6) in (4.5), we finally get the relation

$$F(Z) = F(\xi)\eta(Z). \tag{4.7}$$

Next, substituting X and Z by ξ in (4.3) and in view of (2.3), (2.9) and (2.13), we have

$$\begin{aligned}(\tilde{\nabla}_\xi \tilde{S})(Y, \xi) - [\frac{d\tilde{r}(\xi)}{n}]\eta(Y) =& - [D(\xi) + F(\xi)][\frac{\tilde{r} + 2n(n-1)}{n}]\eta(Y) \\ & - E(Y)[\frac{\tilde{r} + 2n(n-1)}{n}].\end{aligned} \tag{4.8}$$

Also from (2.13), we have

$$(\tilde{\nabla}_\xi \tilde{S})(Y, \xi) = 0. \tag{4.9}$$

By using (4.4) and (4.9) in (4.8), we get

$$E(Y) = E(\xi)\eta(Y). \tag{4.10}$$

In the similar manner, by taking $Y = Z = \xi$ in (4.3), implies

$$\begin{aligned}(\tilde{\nabla}_X \tilde{S})(\xi, \xi) - \frac{d\tilde{r}(X)}{n} =& - D(X)[\frac{\tilde{r} + 2n(n-1)}{n}] \\ & + [D(\xi) - \frac{d\tilde{r}(\xi)}{\tilde{r} + 2n(n-1)}][\frac{\tilde{r} + 2n(n-1)}{n}]\eta(X),\end{aligned} \tag{4.11}$$

on simplification, we have

$$D(X) = \frac{d\tilde{r}(X)}{\tilde{r} + 2n(n-1)} - \frac{d\tilde{r}(\xi)}{\tilde{r} + 2n(n-1)}\eta(X) + D(\xi)\eta(X). \tag{4.12}$$

Summing up of (4.7), (4.10) and (4.12), we obtain

$$D(X) + E(X) + F(X) = \frac{d\tilde{r}(X)}{\tilde{r} + 2n(n-1)}. \tag{4.13}$$

Thus, the proof completes.

5. Special Weakly concircular Ricci-symmetric Para-Sasakian manifold admitting quarter-symmetric metric connection

Motivated by above studies on weakly concircular symmetric and weakly concircular Ricci-symmetric we define and study special weakly concircular Ricci symmetric manifold.

An n-dimensional Riemannian manifold M^n is called a special weakly concircular Ricci symmetric manifold, if it satisfies the following condition

$$(\nabla_X P)(Y,Z) = 2\alpha(X)P(Y,Z) + \alpha(Y)P(X,Z) + \alpha(Z)P(Y,X), \tag{5.1}$$

where α is a 1-form and is defined as

$$\alpha(X) = g(X,\rho),$$

where ρ is an associated vector field.

In view of (5.1), we can define the following definition;

Definition 5.1. A para-Sasakian manifold (M^n, g) $(n > 2)$ is called special weakly concircular Ricci-symmetric with respect to quarter-symmetric metric connection $\tilde{\nabla}$ if the concircular curvature tensor $\bar{P}$ satisfies the following condition

$$(\tilde{\nabla}_X \tilde{\bar{P}})(Y,Z) = 2\alpha(X)\tilde{\bar{P}}(Y,Z) + \alpha(Y)\tilde{\bar{P}}(X,Z) + \alpha(Z)\tilde{\bar{P}}(Y,X), \tag{5.2}$$

for all vector fields X, Y, $Z \in \chi(M^n)$, where α is a 1-form and is defined by

$$\alpha(X) = g(X,\rho), \tag{5.3}$$

where ρ is an associated vector field [24]

Theorem 5.1. If a special weakly concircular Ricci-symmetric para-Sasakian manifold admitting quarter-symmetric metric connection admits cyclic Ricci tensor then the 1-form α must vanish, provided $\{\tilde{r} + 2n(n-1)\} \neq 0$.

Proof: Taking cyclic sum of (5.2), we get

$$\begin{aligned}(\tilde{\nabla}_X \tilde{\bar{P}})(Y,Z) + (\tilde{\nabla}_Y \tilde{\bar{P}})(X,Z) + (\tilde{\nabla}_Z \tilde{\bar{P}})(Y,X) = 4\{\alpha(X)\tilde{\bar{P}}(Y,Z) \\ + \alpha(Y)\tilde{\bar{P}}(X,Z) + \alpha(Z)\tilde{\bar{P}}(Y,X)\}.\end{aligned} \tag{5.4}$$

If M^n admits cyclic Ricci tensor then (5.4) reduces to

$$\alpha(X)\tilde{\bar{P}}(Y,Z) + \alpha(Y)\tilde{\bar{P}}(X,Z) + \alpha(Z)\tilde{\bar{P}}(Y,X) = 0. \tag{5.5}$$

Setting $Z = \xi$ in (5.5), we get

$$\alpha(X)\tilde{\bar{P}}(Y,\xi) + \alpha(Y)\tilde{\bar{P}}(X,\xi) + \alpha(\xi)\tilde{\bar{P}}(Y,X) = 0. \tag{5.6}$$

In view of (2.3), (2.13),(3.2) and (5.3), (5.6) becomes

$$\begin{aligned}&-[\alpha(X)\eta(Y) + \alpha(Y)\eta(X)][\tilde{r} + 2n(n-1)] + \eta(\rho)n[S(Y,X) + 2g(Y,X) \\ &- (n+1)\eta(Y)\eta(X) - 3trace\phi g(\phi Y,X) - \{\tilde{r} + 2n(n-1)\}] = 0.\end{aligned} \tag{5.7}$$

Treating $Y = \xi$ in (5.7) and making use of (2.3), (2.13) and (5.3), we get

$$\alpha(X) + (n+1)\eta(\rho)\eta(X) = 0. \tag{5.8}$$

Again taking $X = \xi$ in (5.8) and using (5.3), we obtain

$$\eta(\rho) = 0. \tag{5.9}$$

Now applying (5.9) in (5.8), we finally see that

$$\alpha(X) = 0. \tag{5.10}$$

Hence, the proof.

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