Quasi Conformal Curvature Tensor on a Lorentzian Para-Sasakian Manifold

Amit Prakash* and Anjana Singh

Azad Institute of Engineering and Technology,
Lucknow, U.P., India
Department of Mathematics
Govt. Degree College, Rewa, M. P.
*e-mail : aproc0185@rediffmail.com
(Received : 22 May, 2009)

Abstract

In this paper, we consider quasi-conformally flat, quasi-conformally conservative and \(\phi\)-quasi conformally flat Lorentzian para-Sasakian manifold. It has also been proved that an Einstein Lorentzian para-Sasakian manifold satisfying the relation \(R(X, Y)\tilde{C} = 0\), where \(\tilde{C}\) is quasi-conformal curvature tensor is locally isometric with a unit sphere.

Keywords : LPS manifold, Quasi conformal curvature tensor, \(\phi\)-quasi conformally flat.

Mathematics Subject Classification 2000 : 53C05, 53C15.

1. Introduction

An \(n\)-dimensional differentiable manifold \(M^n\) is a Lorentzian para-Sasakian (LP-Sasakian) manifold, if it admits a \((1, 1)\)-tensor field \(\phi\), vector field \(\xi\), 1-form \(\eta\) and a Lorentzian metric \(g\) which satisfy

\[
\phi^2 X = X + \eta(X)\xi,
\]

(1.1)

\[
\eta(\xi) = -1,
\]

(1.2)

\[
g(\phi X, \phi Y) = g(X, Y) + \eta(X) \eta(Y),
\]

(1.3)

\[
g(X, \xi) = \eta(X),
\]

(1.4)

\[
(D_X\phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,
\]

(1.5)
and
\[ D_X \xi = \phi X, \]  
(1.6)

for arbitrary vector fields \( X \) and \( Y \); where \( D \) denotes covariant differentiation with respect to \( g \), (Matsumoto, (1989) and Matsumoto and Mihai, (1988)).

In an LP-Sasakian manifold \( M^n \) with structure \((\phi, \xi, \eta, g)\), it is easily seen that

\[ \begin{align*}
(a) \quad & \phi \xi = 0 \\
(b) \quad & \eta(\phi X) = 0 \\
(c) \quad & \text{rank } \phi = (n - 1).
\end{align*} \]  
(1.7)

Let us put
\[ F(X, Y) = g(\phi X, Y). \]  
(1.8)

Then the tensor field \( F \) is symmetric \( (0, 2) \) tensor field

\[ F(X, Y) = F(Y, X), \]  
(1.9)

\[ F(X, Y) = (D_X \eta)(Y), \]  
(1.10)

and
\[ (D_X \eta)(Y) - (D_Y \eta)(X) = 0. \]  
(1.11)

An LP-Sasakian manifold \( M^n \) is said to be Einstein manifold if its Ricci tensor \( S \) is of the form
\[ S(X, Y) = kg(X, Y), \]  
(1.12)

where \( k = (n - 1) \).

An LP-Sasakian manifold \( M^n \) is said to be \( \eta \)-Einstein manifold if its Ricci tensor \( S \) is of the form
\[ S(X, Y) = \alpha g(X, Y) + \beta \eta(X) \eta(Y), \]  
(1.13)

for any vector fields \( X \) and \( Y \), where \( \alpha, \beta \) are functions on \( M^n \).

Let \( M^n \) be an \( n \)-dimensional LP-Sasakian manifold with structure \((\phi, \xi, \eta, g)\). Then we have (Matsumoto and Mihai, (1988) and Mihai, Shaikh and De (1999)).

\[ g(R(X, Y) Z, \xi) = \eta(R(X, Y) Z) = g(Y, Z) \eta(X) - g(X, Z) \eta(Y), \]  
(1.14)

\[ R(\xi, X) Y = g(X, Y) \xi - \eta(Y) X, \]  
(1.15)(a)

\[ R(\xi, X) \xi = X + \eta(X) \xi, \]  
(1.15)(b)
Quasi Conformal Curvature Tensor on a Lorentzian...

\[ R(X, Y) \xi = \eta(Y) X - \eta(X) Y, \quad (1.15)(c) \]

\[ S(X, \xi) = (n - 1) \eta(X), \quad (1.16) \]

\[ S(\phi X, \phi Y) = S(X, Y) + (n - 1) \eta(X) \eta(Y), \quad (1.17) \]

for any vector fields \( X, Y, Z \), where \( R(X, Y) Z \) is the Riemannian curvature tensor of type \((1, 3)\), \( S \) is the Ricci-tensor of type \((0, 2)\), \( Q \) is \((1, 1)\) type Ricci tensor and \( r \) is the scalar curvature, \( g(QX, Y) = S(X, Y) \), for all \( X, Y \).

Quasi-conformal curvature tensor \( \bar{C} \) on a Riemannian manifold \( (M^n, g) \), \((n > 3)\) of type \((1, 3)\) is defined as follows (Yano and Sawaki, (1968)).

\[ \bar{C}(X, Y) Z = aR(X, Y) Z + b[S(Y, Z) X - S(X, Z) Y + g(Y, Z) QX - g(X, Z) QY] \]

\[ - \frac{r}{n} \left[ \frac{a}{(n - 1)} + 2b \right] [g(Y, Z) X - g(X, Z) Y], \quad (1.18) \]

where \( a, b \) are constants such that \( a, b \neq 0 \).

If \( a = 1 \) and \( b = -\frac{1}{(n - 2)} \), then \((1.18)\) takes the form

\[ \bar{C}(X, Y) Z = R(X, Y) Z - \frac{1}{(n - 2)} [S(Y, Z) X - S(X, Z) Y + g(Y, Z) QX \]

\[ - g(X, Z) QY] + \frac{r}{(n - 1)(n - 2)} [g(Y, Z) X - g(X, Z) Y] = C(X, Y) Z, \]

where \( C \) is the conformal curvature tensor. Thus the conformal curvature tensor \( C \) is a particular case of the tensor \( \bar{C} \). For this reason \( \bar{C} \) is called the quasi-conformal curvature tensor.

Let \( L(X, Y) = S(X, Y) - \frac{r}{2(n - 1)} g(X, Y), \quad (1.19) \)

and \( g(NX, Y) = L(X, Y), \quad (1.20) \)

where \( L \) and \( N \) are tensor field of type \((0, 2)\) and \((1, 1)\) respectively.

From \((1.19)\) and \((1.20)\), we get

\[ N(X) = QX - \frac{r}{2(n - 1)} X. \quad (1.21) \]
Using (1.19) and (1.20), we can write (1.18) as follows

\[ \tilde{C} (X, Y) Z = a \text{R}(X, Y) Z + b[L(Y, Z) X - L(X, Z) Y + g(Y, Z) NX - g(X, Z) NY] \]
\[ - \lambda \frac{r}{n} [g(Y, Z) X - g(X, Z) Y], \]  
(1.22)

where \( \lambda = \frac{a + (n - 2)b}{n(n - 1)} \).

2. An Einstein LP-Sasakian manifold satisfying \( \tilde{C}(X, Y) Z = 0 \).

In this section we assume that \( \tilde{C}(X, Y) Z = 0 \).

Then from (1.18), we get

\[ a \text{R}(X, Y) Z = -b[S(Y, Z) X - S(X, Z) Y + g(Y, Z) QX - g(X, Z) QY] \]
\[ + \frac{r}{n} \left[ \frac{a}{(n - 1)} + 2b \right] [g(Y, Z) X - g(X, Z) Y], \]
(2.1)
or

\[ a' \text{R}(X, Y, Z, W) = -b[S(Y, Z) g(X, W) - S(X, Z) g(Y, W) + g(Y, Z) S(X, W) \]
\[ - g(X, Z) S(Y, W)] + \frac{r}{n} \left[ \frac{a}{(n - 1)} + 2b \right] [g(Y, Z) g(X, W) \]
\[ - g(X, Z) g(Y, W)], \]
(2.2)

where \( a' \text{R}(X, Y, Z, W) = g(R(X, Y) Z, W) \).

Putting \( X = W = \xi \) in (2.2) and using (1.12), we get

\[ [a + 2b(n - 1)] [r - n(n - 1)] g(\phi Y, \phi Z) = 0. \]
(2.3)

Thus we see that \( g(\phi Y, \phi Z) \neq 0 \).

Hence from (2.3), we get \( r = n(n - 1) \), provided \( a + 2b(n - 1) \neq 0 \).

Hence, we can state the following theorem:

**Theorem 1.** An Einstein LP-Sasakian manifold satisfying the condition \( \tilde{C}(X, Y) Z = 0 \), has constant curvature \( r = n(n - 1) \), provided \( a + 2b(n - 1) \neq 0 \).
Contracting equation (1.18) with respect to $X$, we get

\[
(S^1_\mathcal{C})(Y, Z) = a S(Y, Z) + b [S(Y, Z) n - S(X, Z) + g(Y, Z) r - S(Y, Z)] \\
- \frac{r}{n} \left[ \frac{a}{(n - 1)} + 2b \right] [g(Y, Z) n - g(Y, Z)] \\
= [a + (n - 2) b] [S(Y, Z) - \frac{r}{n} g(Y, Z)],
\]

(2.4)

where $(S^1_\mathcal{C})(Y, Z)$ is the contraction of $\mathcal{C}(X, Y) Z$ with respect to $X$.

If $(S^1_\mathcal{C})(Y, Z) = 0$, we get

\[
S(Y, Z) = \frac{r}{n} g(Y, Z), \quad \text{provided} \quad a + (n - 2) b \neq 0.
\]

(2.5)

Hence from (2.2) it follows that

\[
a [\mathcal{R}(X, Y, Z, W) - \frac{r}{n (n - 1)} \{g(Y, Z) g(X, W) - g(X, Z) g(Y, W)\}] = 0.
\]

(2.6)

Therefore, from (2.6), we get

\[
\mathcal{R}(X, Y, Z, W) = \frac{r}{n (n - 1)} [g(Y, Z) g(X, W) - g(X, Z) g(Y, W)],
\]

(2.7)

provided $a \neq 0$.

Hence, we can state the following theorem:

**Theorem 2.** A quasi-conformally flat $(M^n, g)$ $(n > 3)$, LP-Sasakian manifold satisfying $(S^1_\mathcal{C})(Y, Z) = 0$ is a manifold of constant curvature provided $a \neq 0$.

Using (2.5) and (2.7) in (2.2), we get

\[
\mathcal{R}(X, Y, Z, W) = 0.
\]

From this it follows that

\[
\mathcal{C}(X, Y) Z = 0.
\]

Hence the manifold is quasi-conformally flat.

Hence, we can state the following theorem:
\textbf{Theorem 3.} An LP-Sasakian manifold \((M^n, g)\) \((n > 3)\) satisfying \((S^1 \tilde{C})(Y, Z) = 0\), of constant curvature is quasi-conformally flat.

\section{Einstein LP-Sasakian manifold satisfying \((\text{div} \tilde{C})(X, Y) Z = 0\).}

\textbf{Definition.} A manifold \((M^n, g)\) \((n > 3)\) is called quasi-conformally conservative if (Hicks, N. J. (1969)), \(\text{div} \tilde{C} = 0\).

In this section we assume that

\[
\text{div} \tilde{C} = 0, \tag{3.1}
\]

where \(\text{div}\) denotes divergence.

From (1.21), we get

\[
N = Q - \frac{rI}{2(n-1)}.
\]

Hence

\[
\text{div} N = \text{div} Q - \frac{\text{dr}}{2(n-1)}. \tag{3.2}
\]

But \(\text{div} Q = \frac{1}{2} \text{dr}\), therefore

\[
\text{div} N = \frac{(n-2)}{2(n-1)} \text{dr}. \tag{3.3}
\]

Now differentiating (1.22) covariantly, we get

\[
(D_w \tilde{C})(X, Y) Z = a(D_w R)(X, Y) Z + b[(D_w L)(Y, Z) X - (D_w L)(X, Z) Y
+ g(Y, Z) (D_w N)(X) - g(X, Z) (D_w N)(Y)]
- \lambda (D_w r) [g(Y, Z) X - g(X, Z) Y], \tag{3.4}
\]

which gives on contraction

\[
(\text{div} \tilde{C})(X, Y) Z = a(\text{div} R)(X, Y) Z + b[(D_x L)(Y, Z) - (D_y L)(X, Z)]
+ \left[\frac{(n-2)b}{2(n-1)} - \lambda\right][g(Y, Z) \text{dr}(X) - g(X, Z) \text{dr}(Y)]. \tag{3.5}
\]

We have (Eisenhart, L. P. (1926)).
\((\text{div } R) (X, Y) Z = (D_X S) (Y, Z) - (D_Y S) (X, Z)\)
\[= (D_X L) (Y, Z) - (D_Y L) (X, Z) + \frac{1}{2(n-1)} [g(Y, Z) \, \text{dr}(X) - g(X, Z) \, \text{dr}(Y)].\]  

Hence (3.5) takes the form
\[
(\text{div } \bar{C}) (X, Y) Z = (a + b) [(D_X L) (Y, Z) - (D_Y L) (X, Z)]
+ \frac{(n-2) [a + b(n-2)]}{2n(n-1)} [g(Y, Z) \, \text{dr}(X) - g(X, Z) \, \text{dr}(Y)].
\]  

If LP-Sasakian manifold is an Einstein manifold, then we have

\[(D_X L) (Y, Z) - (D_Y L) (X, Z) = 0,\]

which gives from (3.7) that

\[
(\text{div } \bar{C}) (X, Y) Z = \frac{(n-2) [a + b(n-2)]}{2n(n-1)} [g(Y, Z) \, \text{dr}(X) - g(X, Z) \, \text{dr}(Y)].
\]  

Hence if \(\text{div } \bar{C} = 0\), then \(g(Y, Z) \, \text{dr}(X) - g(X, Z) \, \text{dr}(Y) = 0\), provided \(a + (n-2) \, b \neq 0\). Consequently \(r\) is constant. Again if \(r\) is constant then from (3.8) it follows that \((\text{div } \bar{C}) (X, Y, Z) = 0\).

Hence, we can state the following theorem:

**Theorem 4.** An Einstein LP-Sasakian manifold \((M^n, g) \ (n > 3)\) is quasi conformally conservative if and only if the scalar curvature is constant, provided \(a + (n-2) \, b \neq 0\).

4. **LP-Sasakian manifold satisfying** \(\phi^2 \bar{C} (\phi X, \phi Y) \phi Z = 0\).

**Definition.** A differentiable manifold \((M^n, g) \ (n > 3)\), satisfying the condition \(\phi^2 \bar{C} (\phi X, \phi Y) \phi Z = 0\) is called f-quasi conformally flat (Cabreizo, Fernandez, Fernandez and Zhen (1999)).

In this section we assume that LP-Sasakian manifold \((M^n, g) \ (n > 3)\), is \(\phi\)-quasi conformally flat, then \(\phi^2 \bar{C} (\phi X, \phi Y) \phi Z = 0\) implies
\[ g(\mathcal{C} (\phi X, \phi Y, \phi Z, \phi W) = 0, \] (4.1)

for any vector fields \( X, Y, Z, W \).

So by the use of (1.18), \( \phi \)-quasi conformally flat means

\[
a' R(\phi X, \phi Y, \phi Z, \phi W) = -b \left[ S(\phi Y, \phi Z) g(\phi X, \phi W) - S(\phi X, \phi Z) g(\phi Y, \phi W) \right] + \frac{r}{n} \left[ \frac{a}{(n - 1)} + 2b \right] \\
+ g(\phi Y, \phi Z) S(\phi X, \phi W) - g(\phi X, \phi Z) S(\phi Y, \phi W) \right] + \frac{r}{n} \left[ \frac{a}{(n - 1)} + 2b \right] \\
[ g(\phi Y, \phi Z) g(\phi X, \phi W) - g(\phi X, \phi Z) g(\phi Y, \phi W) ] , \] (4.2)

where \( R(\phi X, \phi Y, \phi Z, \phi W) = g(R(\phi X, \phi Y, \phi Z, \phi W) \).\]

Let \( \{ e_1, e_2, \ldots, e_{n - 1}, \xi \} \) be a local orthogonal basis of vector fields in \( M^n \) by using the fact that \( \{ \phi e_1, \phi e_2, \ldots, \phi e_{n - 1}, \xi \} \) is also a local orthonormal basis, if we put \( X = W = e_i \) in (4.2) and sum up with respect to \( i \), then we have

\[
\sum_{i=1}^{(n-1)} a' R(\phi e_i, \phi Y, \phi Z, \phi e_i) = -b \sum_{i=1}^{(n-1)} \left[ S(\phi Y, \phi Z) g(\phi e_i, \phi e_i) - S(\phi e_i, \phi Z) g(\phi Y, \phi e_i) \right] \\
+ g(\phi Y, \phi Z) S(\phi e_i, \phi e_i) - g(\phi e_i, \phi Z) S(\phi Y, \phi e_i) \right] + \frac{r}{n} \left[ \frac{a}{(n - 1)} + 2b \right] \\
\sum_{i=1}^{(n-1)} \left[ g(\phi Y, \phi Z) g(\phi e_i, \phi e_i) - g(\phi e_i, \phi Z) g(\phi Y, \phi e_i) \right] . \] (4.3)

On an LP-Sasakian manifold, we have (Özgür (2003))

\[
\sum_{i=1}^{(n-1)} a' R(\phi e_i, \phi Y, \phi Z, \phi e_i) = S(\phi Y, \phi Z) + g(\phi Y, \phi Z), \] (4.4)

\[
\sum_{i=1}^{(n-1)} S (\phi e_i, \phi e_i) = r + n - 1, \] (4.5)

\[
\sum_{i=1}^{(n-1)} g(\phi e_i, \phi Z) S (\phi Y, \phi e_i) = S(\phi Y, \phi Z), \] (4.6)
\[
\sum_{i=1}^{n-1} g(\phi e_i, \phi e_i) = n + 1, \quad (4.7)
\]
\[
\sum_{i=1}^{n-1} g(\phi e_i, \phi Z) g(\phi Y, \phi e_i) = g(\phi Y, \phi Z). \quad (4.8)
\]

So by virtue of (4.4) - i. -, the equation (4.3) takes the form
\[
[a + b (n - 1)] S(\phi Y, \phi Z) = \left[ \frac{ar}{(n - 1)} + br - bn + b - a \right] g(\phi Y, \phi Z). \quad (4.9)
\]

Then by making use of (1.3) and (1.17), the equation (4.9) takes the form
\[
[a + b(n - 1)] [S(Y, Z) - \left( \frac{r}{n-1} - 1 \right) g(Y, Z)] - \left( \frac{r}{n-1} - n \right) \eta(Y) \eta(Z) = 0,
\]
which gives
\[
S(Y, Z) = \left( \frac{r}{n-1} - 1 \right) g(Y, Z) + \left( \frac{r}{n-1} - n \right) \eta(Y) \eta(Z),
\]
provided \( a + (n - 1) b \neq 0 \).

Which shows that \( M^0 \) is an \( \eta \)-Einstein manifold, provided \( a + (n - 1) b \neq 0 \), with constants \( \alpha \) and \( \beta \) are same as in \( \eta \)-Einstein manifold of an LP-Sasakian manifold, given by \( \alpha = \left( \frac{r}{n-1} - 1 \right) \) and \( \beta = \left( \frac{r}{n-1} - n \right) \).

Hence, we can state the following theorem:

**Theorem 5.** Let \( M^0 \) be an \( n \)-dimensional \( (n > 3) \), \( \phi \)-quasi conformally flat LP-Sasakian manifold. Then \( M^0 \) is an \( \eta \)-Einstein manifold, provided \( a + (n - 1) b \neq 0 \), with constants \( \alpha = \left( \frac{r}{n-1} - 1 \right) \) and \( \beta = \left( \frac{r}{n-1} - n \right) \).

5. **An Einstein LP-Sasakian manifold satisfying** \( R(X, Y)\bar{\nabla} = 0 \).

In this section we assume that \( R(X, Y)\bar{\nabla} (U, V) W = 0 \). \quad (5.1)

Let the Riemannian manifold \( M^0 \) be an Einstein manifold, then (1.18) gives
\[ \dot{C}(X, Y, Z, W) = a'R(X, Y, Z, W) + \left[ 2bk - \frac{r}{n} \left\{ \frac{a}{(n-1)} + 2b \right\} \right] \left[ g(Y, Z) g(X, W) - g(X, Z) g(Y, W) \right], \] (5.2)

where \[ \dot{C}(X, Y, Z, W) = g(C(X, Y) Z, W). \]

Putting \( W = \xi \) in (5.2) and using (1.14), we get

\[ \eta(\ddot{C}(X, Y) Z) = \left[ a + 2bk - \frac{r}{n} \left\{ \frac{a}{(n-1)} + 2b \right\} \right] \left[ g(Y, Z) \eta(X) - g(X, Z) \eta(Y) \right]. \] (5.3)

Taking \( X = \xi \) in (5.3), we get

\[ \eta(\ddot{C}(\xi, Y) Z) = \left[ a + 2bk - \frac{r}{n} \left\{ \frac{a}{(n-1)} + 2b \right\} \right] [- g(Y, Z) - \eta(Y) \eta(Z)]. \] (5.4)

\[ \eta(\ddot{C}(X, Y) \xi) = 0. \] (5.5)

Now,

\[ (R(X, Y) \ddot{C})(U, V) W = R(X, Y) \ddot{C}(U, V) W - \ddot{C}(R(X, Y) U, V) W \]

\[ - \ddot{C}(U, R(X, Y) V) W - \ddot{C}(U, V) R(X, Y) W. \] (5.6)

In view of (5.1), we get

\[ R(X,Y)\ddot{C}(U,V)W - \ddot{C}(R(X,Y)U, V)W - \ddot{C}(U,R(X,Y)V)W - \ddot{C}(U,V)R(X,Y)W = 0. \]

Putting \( X = \xi \) and taking the inner product of the above equation with \( \xi \), we get

\[ g(R(\xi, Y) \ddot{C}(U, V) W, \xi) - g(\ddot{C}(R(\xi, Y) U, V) W, \xi) - g(\ddot{C}(U, R(\xi, Y) V) W, \xi) \]

\[ - g(\ddot{C}(U, V) R(\xi, Y) W, \xi) = 0. \]

From this it follows that

\[ - \dot{C}(U, V, W, Y) - \eta(Y) \eta(\dot{C}(U, V) W) + \eta(U) \eta(\dot{C}(Y, V) W) + \eta(V) \eta(\dot{C}(U, Y) W) \]

\[ + \eta(W) \eta(\dot{C}(U, Y) Y) - g(Y, U) \eta(\dot{C}(\xi, V) W) - g(Y, V) \eta(\dot{C}(U, \xi) W) \]

\[ - g(Y, W) \eta(\dot{C}(U, V) \xi) = 0. \] (5.7)
Putting \( Y = U \) in (5.7), we get

\[
- \bar{\mathcal{C}}(U, V, W, U) - \eta(U) \eta(\bar{\mathcal{C}}(U, V) W) + \eta(U) \eta(\bar{\mathcal{C}}(U, V) W) + \eta(V) \eta(\bar{\mathcal{C}}(U, V) W) \\
+ \eta(V) \eta(\bar{\mathcal{C}}(U, V) W) - g(U, U) \eta(\bar{\mathcal{C}}(\xi, V) W) - g(U, V) \eta(\bar{\mathcal{C}}(U, \xi) W) \\
- g(U, V) \eta(\bar{\mathcal{C}}(U, \xi) W) - g(U, W) \eta(\bar{\mathcal{C}}(U, V) \xi) = 0.
\] (5.8)

Let \( \{e_i\}, i = 1, 2, 3, \ldots, n \) be an orthogonal basis of the tangent space at any point. Then the sum for \( 1 \leq i \leq n \) of relation (5.8), for \( U = e_i \), gives

\[
\eta(\bar{\mathcal{C}}(\xi, V) W) = \frac{1}{(n-1)} \left[ - a S(V, W) - \left( 2bk - \frac{r}{n} \left( \frac{a}{n-1} + 2b \right) \right) (n-1) g(V, W) \\
- \left( a + 2bk - \frac{r}{n} \left( \frac{a}{n-1} + 2b \right) \right) (n-1) \eta(V) \eta(W) \right].
\] (5.9)

Using (5.3) and (5.9), it follows from (5.7) that

\[
\bar{\mathcal{C}}(U, V, W, Y) = \left[ 2bk - \frac{r}{n} \left( \frac{a}{n-1} + 2b \right) \right] [g(V, W) g(Y, U) - g(V, Y) g(U, W)] \\
+ \frac{a}{(n-1)} [S(V, W) g(Y, U) - S(U, W) g(V, Y)].
\] (5.10)

Using (1.12) in (5.10), we get

\[
\bar{\mathcal{C}}(U, V, W, Y) = \left[ a + 2bk - \frac{r}{n} \left( \frac{a}{n-1} + 2b \right) \right] [g(V, W) g(Y, U) - g(V, Y) g(U, W)].
\] (5.11)

From equation (5.2) and (5.11), we get

\[ \bar{\text{R}}(U, V, W, Y) = g(V, W) g(Y, U) - g(U, W) g(V, Y), \] provided \( a \neq 0 \).

Hence, we can state the following theorem:

**Theorem 6.** If in an Einstein LP-Sasakian manifold, the relation \( \bar{\text{R}}(X, Y) \bar{\mathcal{C}} = 0 \) holds, then it is locally isometric with a unit sphere, provided \( a \neq 0 \).
References


