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Probing an exact universe with recent H(z) and Pantheon data

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In this paper, we have investigated an exact solution of Einstein's field equation of isotropic and homogeneous universe. We have performed χ^2 test to obtain the best fit value of model parameters of derived model with its observed values. It is obtained that the best fit values of Hubble constant and density parameters are $H_0 = 68.13 \pm 1.17$, $(\Omega_m)_0 = 0.27 \pm 0.005$ and $(\Omega_\Lambda)_0 = 0.718 \pm 0.015$ by bounding the derived model with latest $H(z)$ data while with Pantheon data, its values are $H_0 = 71.47 \pm 0.53$, $(\Omega_m)_0 = 0.276 \pm 0.006$ and $(\Omega_\Lambda)_0 = 0.74 \pm 0.01$. The dynamics of deceleration parameter shows that in the derived model, the universe was in decelerating phase for the transition redshift $z_t > 0.723$. At $z_t = 0.723$, the present universe has entered in accelerating phase of expansion. The age of current universe is obtained as 13.89 ± 0.017 Gyrs which is in good consistency with its value observed by Plank collaboration results and WMAP observations.

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1. Introduction:

Cosmology is a science concerning to the physical universe in which we study how it came into existence in its present form after going over so many drastic evolutions right from the big bang singularity in its very birth. It is well known that the scenario has drastically changed during the last three decades. Before, researchers were working on dust matter dominated decelerating universe considering the Einstein de Sitter model as a standard one. Supernova Cosmology Project Team [1,2] and the High Redshift Search Team [3] reveal that the distant supernovae are fainter and they are more away from us than it is expected. The study[4,5] also points to the larger value of critical density. So a substantial amount of energy component apart from the baryon matter density must be present in the universe. More over the missing energy should speed up the cosmic expansion of he universe in order to explain the observed supernovae red shift- magnitude relation ship. This is possible only when the so called missing energy which is generally known as dark energy(DE) have negative pressure to counteract gravitation pressure of barionic matter. Observations tells us that at present DE is dominant and covers nearly 70% of the total energy content. The Einstein's cosmological constant Λ as a source of energy has repulsive character, so it is a natural choice for dark energy.The standard Friedman Lemaitre Robertson Walker (FLRW) model of the universe with cosmological constant as a source of dark energy is often known as Λ-CDM cosmological model [6] [7]. It is also called concordance model. Basically, the standard FRW model represents decelerating universe but presence of cosmological constant as a source and its specific value makes the model accelerating. It is found that the Λ-CDM model is in good agreement with latest observations [8] [9] [10] [11]. Wilkinson Microwave Anisotropy Probe [6] and Hubble Key Project [12] explored that our universe is specially dust filled nearly flat. The Λ-CDM cosmological model provides concept of the two component baryon matter and DE(dark energy) density parameters Ω_m and Ω_Λ, which are related through $\Omega_\Lambda + \Omega_m = 1$.

Seeing the success of Λ-CDM model on observational ground, it is desirable to investigate its roll in early radiation filled universe. For this purpose, we have developed a model of the universe in which baryon matter also has pressure. The attraction in the model is that we have found exact hyperbolic solution for scale factor which shows transition from deceleration to acceleration. It is rightly said that "All of observational cosmology is the search for two numbers: Hubble (HP) and deceleration parameters(DP) H_0 and q_0[13]." In the present scenario higher derivatives of scale factor such as jerk parameter j_0, s_0 and l_0 do play important role in state finder diagnostic[14,15]. A successful cosmological model will be one in which these parameters fits best with the observational inputs. Keeping this as our motto, We have performed χ^2 test to obtain the best fit value of model parameters of derived model with its observed values. It is obtained that the best fit values of Hubble constant and density parameters are $H_0 = 68.13 \pm 1.17$, $(\Omega_m)_0 = 0.27 \pm 0.005$ and $(\Omega_\Lambda)_0 = 0.718 \pm 0.015$ by bounding the derived model with latest $H(z)$ data while with Pantheon data, its values are $H_0 = 71.47 \pm 0.53$, $(\Omega_m)_0 = 0.276 \pm 0.006$ and $(\Omega_\Lambda)_0 = 0.74 \pm 0.01$. The dynamics of deceleration parameter shows that in the derived model, the universe was in decelerating phase for the transition red-shift $z_t > 0.723$. At $z_t = 0.723$, the present universe has entered in accelerating phase of expansion. The age of current universe is obtained as 13.89 ± 0.017 Gyrs which is in good consistency with its value observed by Plank collaboration results[16] and WMAP observations.

2. The model and basic formalism

The isotropic and homogeneous gravitational field is read as

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{1}$$

where $a(t)$ is the scale factor.
The Einstein's field equation with cosmological constant (Λ) is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} - \Lambda g_{ij} = 8\pi G T_{ij} \tag{2}$$

where R is the Ricci scalar and other symbols have their usual meaning.
The energy-momentum tensor (T_{ij}) of perfect fluid is read as

$$T_{ij} = (\rho + p)v_i v_j - p g_{ij} \tag{3}$$

where v^i is four velocity vector satisfying $v^i v_i = 1$.
In equation (3), p and ρ are the isotropic pressure and energy density of the fluid under consideration.

Solving (2) with space-time (1), we obtain the following system of equations

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p + \Lambda \tag{4}$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G \rho + \Lambda \tag{5}$$

The barotropic equation of state for perfect fluid is read as

$$p = \omega \rho \tag{6}$$

where $0 \leq \omega \leq 1$ is equation of state parameter.
Using (4) and (5), the equation (6) becomes:

$$\frac{2\,\ddot{a}}{a} + (1 + 3\,\omega)\left(\frac{\dot{a}}{a}\right)^2 = (1 + \omega)\,\Lambda. \tag{7}$$

The general solution of equation (7) is obtained as

$$a(t) = m\,\sinh\left[\Lambda_0\,(1+\omega)\,t + t_0\right]^{\overline{\frac{2}{3\,(1+\omega)}}} \tag{8}$$

and

$$H^2 = \frac{\Lambda}{3}\,\coth\left[\Lambda_0\,(1+\omega)\,t + t_0\right]^2 \tag{9}$$

where $\Lambda_0 = \sqrt{\frac{3\Lambda}{4}}$ and m and t_0 are the arbitrary constant of integration.
Using $\dot{z} = -(1+z)H$, Eq.(7) is transformed to

$$2(z+1)HH' - 3(\omega+1)H^2 + \Lambda(\omega+1) = 0,$$

where dash represents differentiation with respect to red shift z. This is linear diff. eqn. in H^2, so its solution is as follows

$$H = H_0\sqrt{\left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right)}, \tag{10}$$

where

$$(\Omega_m)_0 = 1 - \frac{\Lambda}{3H_0^2} \ \ \& \ \ (\Omega_\Lambda)_0 = \frac{\Lambda}{3H_0^2}$$

$$\therefore \ \ (\Omega_m)_0 + (\Omega_\Lambda)_0 = 1$$

Thus, the expression for luminosity distance (D_L) and Distance Modulus (μ) are obtained as

$$D_L = \frac{(1+z)}{H_0}\int_0^z \frac{dz}{\sqrt{\left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right)}}, \tag{11}$$

and

$$\mu = 25 + 5log_{10}\left(\frac{(1+z)}{H_0}\int_0^z \frac{dz}{\sqrt{\left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right)}}\right) \tag{12}$$

3. Observational constraints on the model parameters

In this section, we describe $H(z)$ and Pantheon observational data and the statistical methodological analysis for constraining various model parameters .

- **Observational Hubble Data (OHD)**: We have take over 46 $H(z)$ observational datapoints in the range of $0 \le z \le 2.36$, dominated from cosmic chronometric technique. These all 46 $H(z)$ datapoints are compiled in table I of Ref. [17].

- **Pantheon data**: We use the Pantheon compilation [18] which includes 1048 SNIa apparent magnitude measurements including 276 SNIa ($0.03 < z < 0.65$) investigated by the Pan-STARRS1 Medium Deep Survey and SNIa distance estimates from SDSS, SNLS and low-z HST samples.

Now, we define χ^2 for H(z) points as following:

$$\chi^2_{H(z)} = \sum_{i=1}^{46}\left[\frac{H_{th}(z_i) - H_{obs}(z_i)}{\sigma_i}\right]^2 \tag{13}$$

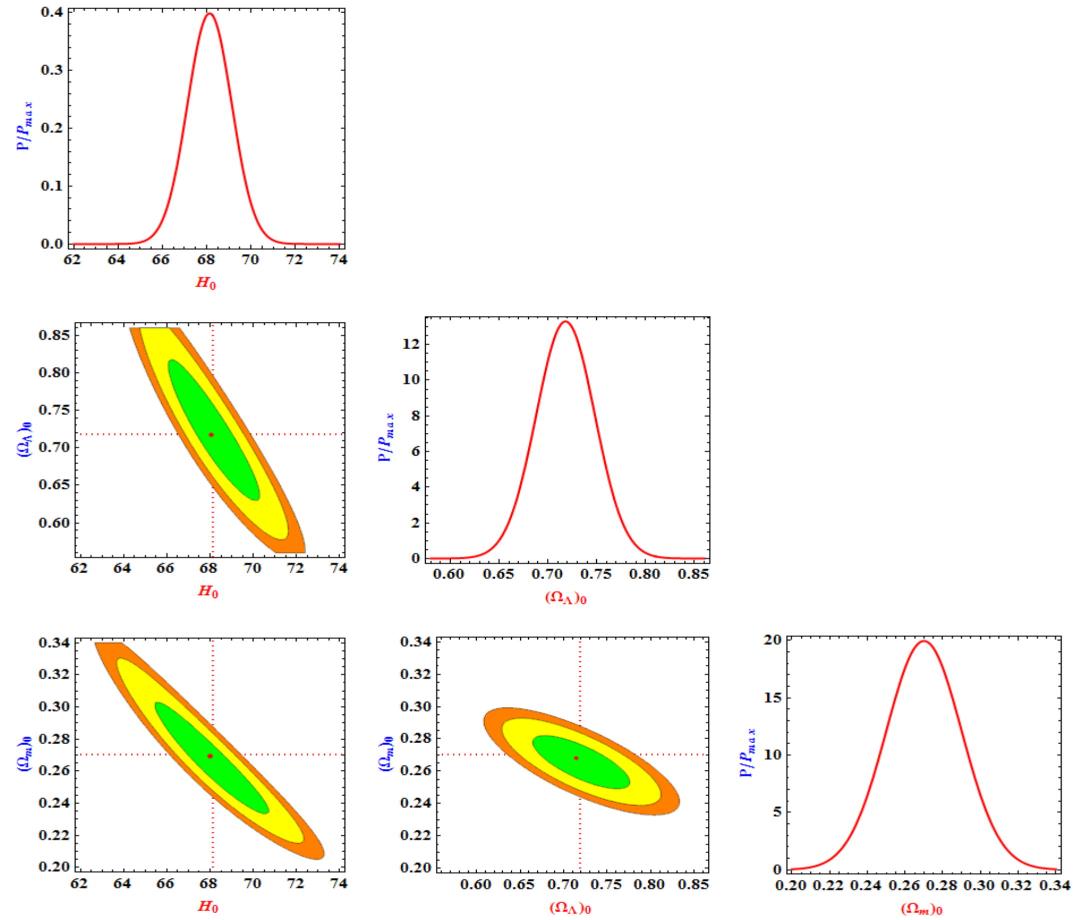

Fig. 1. One-dimensional marginalized distributions and two dimensional contours at 1σ, 2σ and 3σ confidence regions by bounding our model with latest 46 observational Hubble data.

where $H_{obs}(z_i)$ is the observed value of Hubble parameter with errors σ_i and $H_{th}(z_i)$ is the theoretical values obtained from bounding equation (13) with 46 H(z) points.

Similarly for Pantheon data, we have

$$\chi^2_{Pantheon} = \sum_{i=1}^{1048} \left[\frac{\mu_{th}(z_i) - \mu_{obs}(z_i)}{\sigma_i} \right]^2 \tag{14}$$

where $\mu_{obs}(z_i)$ is the observed value of distance modulus with deviation σ_i and $\mu_{th}(z_i)$ is its theoretical values obtained from bounding equation (14) with Pantheon data.

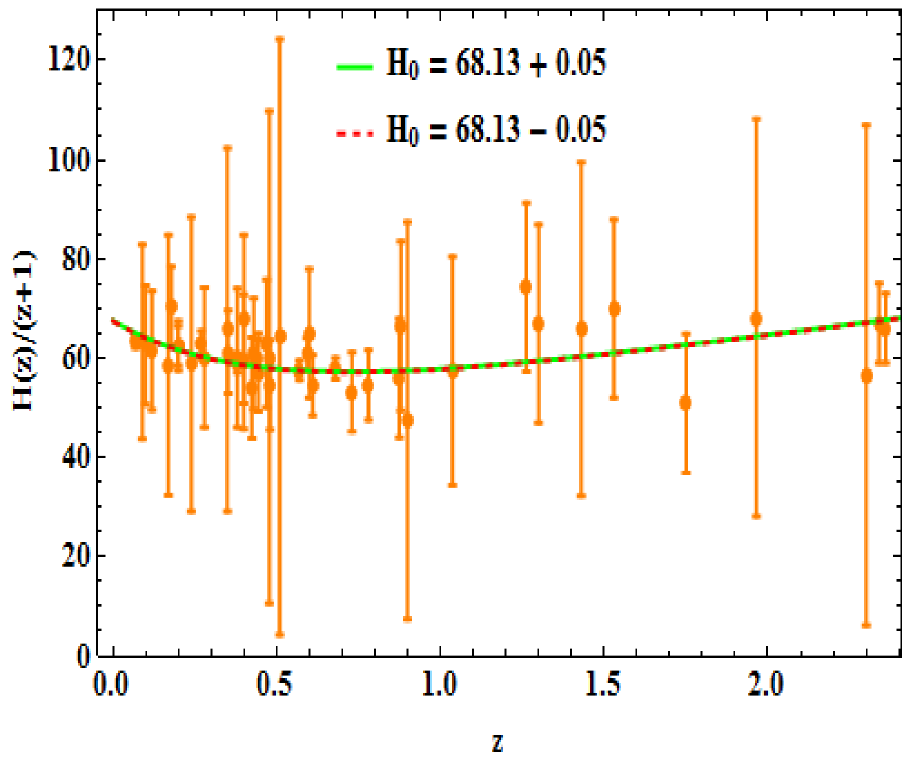

Fig. 2. The plot of Hubble rate $H(z)/(1+z)$ versus z for $H_0 = 68.13 \pm 0.05\ km\ s^{-1}\ Mpc^{-1}$, $(\Omega_m)_0 = 0.27$, $(\Omega_\Lambda)_0 = 0.718$ and $\omega = 0.01$.

Figure 1 exhibits one-dimensional marginalized distributions and two dimensional contours at 1σ, 2σ and 3σ confidence levels by bounding our model with latest 46 observational Hubble data. The summary of statistical analysis is as follows: $H_0 = 68.13 \pm 1.17$, $(\Omega_m)_0 = 0.27 \pm 0.005$ and $(\Omega_\Lambda)_0 = 0.718 \pm 0.015$. Figure 2 depicts the best fit curve of Hubble rate with red-shift of derived model with OHD.

Figure 3 depicts one-dimensional marginalized distributions and two dimensional contours at 1σ, 2σ and 3σ confidence regions by bounding our model with Pantheon data. The summary of statistical analysis is as follows: $H_0 = 71.47 \pm 0.53$, $(\Omega_m)_0 = 0.276 \pm 0.006$ and $(\Omega_\Lambda)_0 = 0.74 \pm 0.01$.

4. Physical properties of the model

4.1. *Deceleration parameter*

From Eq.(7), the deceleration parameter is given by

$$q = \frac{(1+3\omega)}{2} - \frac{3(1+\omega)(\Omega_\Lambda)_0}{2\left[(\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right]} \tag{15}$$

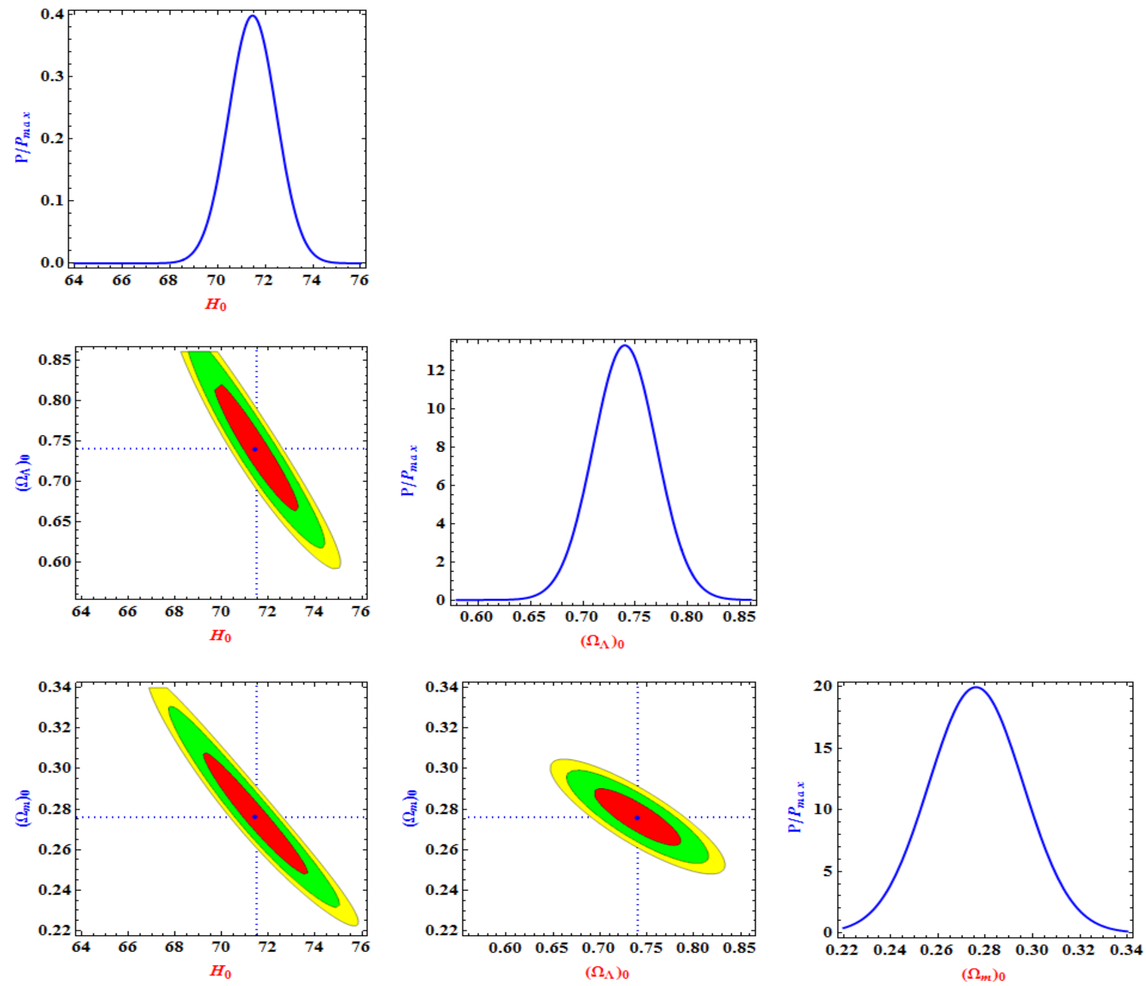

Fig. 3. One-dimensional marginalized distributions and two dimensional contours at 1σ, 2σ and 3σ confidence regions by bounding our model with Pantheon data

The transitioning behavior of deceleration parameter (q) versus red-shift (z) is shown in Figure 4. The transitional red-shift is obtained as $z_t = 0.723$. The sign of q indicates weather the model inflates or not. A positive sign of q corresponds the decelerated expansion of universe while negative sign of q describes the present accelerated expansion of the universe.Here, it is important to mention that the value of transition redshift obtained for our model is comparable to the values obtained from different models[21,22,23]. The present value of q is obtained by putting $z = 0$ in equation (15) i. e.

$$q_0 = \frac{(1+3\omega)}{2} - \frac{3(1+\omega)(\Omega_\Lambda)_0}{2\left[(\Omega_m)_0 + (\Omega_\Lambda)_0\right]} \tag{16}$$

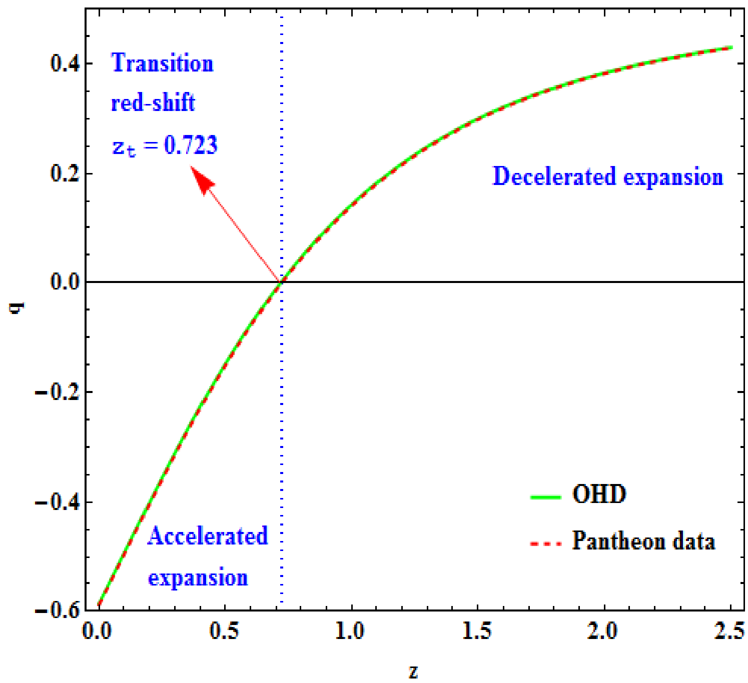

Fig. 4. The behavior of q versus z.

4.2. *Age of universe*

The age of the universe is computed as

$$dt = -\frac{dz}{(1+z)H(z)} \Rightarrow \int_t^{t_0} dt = \int_0^z \frac{1}{(1+z)H(z)} dz \tag{17}$$

$$\therefore \quad t_0 = lim_{z\to\infty} \int_0^z \frac{dz}{(1+z)H_0\sqrt{\left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right)}} \tag{18}$$

Integrating equation (17), we obtain

$$H_0 t_0 = 0.9734$$

Therefore the present age of the universe is estimated as $t_0 = 0.9734 H_0^{-1} = 13.89 \pm 0.01$ Gyrs. Thus the age of the universe in derived model is nicely tally with the age observed by WMAP observations [19] and Plank collaboration [19]. Thus, the derived model has pretty consistency with astrophysical observations. Figure 5 exhibits the plot of $H_0(t_0 - t)$ versus z. From Fig. 5, we observe that at $z = 0$, $H_0(t_0 - t)$ becomes null. Therefore, at $z = 0$, $t = t_0$.

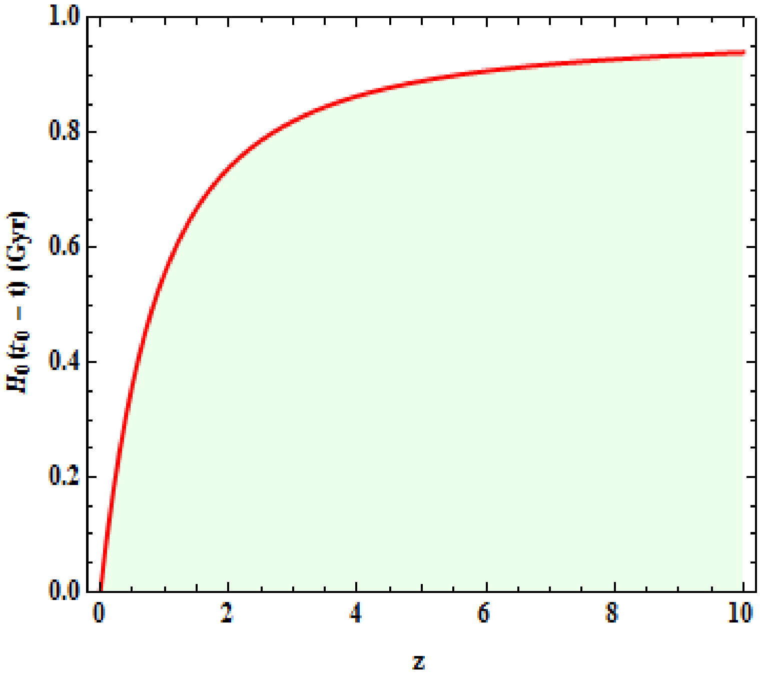

Fig. 5. The plot of $H_0(t_0 - t)$ versus z for $\omega = 0.01$, $(\Omega_m)_0 = 0.276$ and $(\Omega_\Lambda)_0 = 0.74$.

4.3. *Jerk parameter"j"*

The jerk parameter 'j' is defind as $j = \frac{a'''}{aH^3}$. It is related to the third order of scale factor 'a'. In our model it is calculated as

$$j = 1 - \frac{9\omega(\omega+1)\left((\Omega_{\mathrm{m}})_0 (z+1)^{3(\omega+1)}\right)}{2(\Omega_\Lambda)_0}.$$

This clearly shows that the present value of jerk is almost one as EoS parameter for matter ω is very small at present due to dust dominated phase of universe. The attached figure describe the evolution of jerk over red shift. The present value of jerk is obtained as $j_0 = 0.99807$ corresponding our estimated values of parameters $(\Omega_m)_0$ and $(\Omega_\Lambda)_0$.

4.4. *Effective energy density and effective pressure*

Effective energy density and pressure(DP) corresponds over all density parameter which includes both baryon and Λ energy density parameter. Thus, we define

$$\rho_{eff} = \rho_m + \rho_\Lambda, \ p_{eff} = p_m + p_\Lambda, \ p_\Lambda = -\rho_\Lambda = -\Lambda/8\pi G$$

Therefore, the field equations (4) and (5) recast as

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G \, p_{eff}, \ p_{eff} = p_m - \Lambda/8\pi G \tag{19}$$

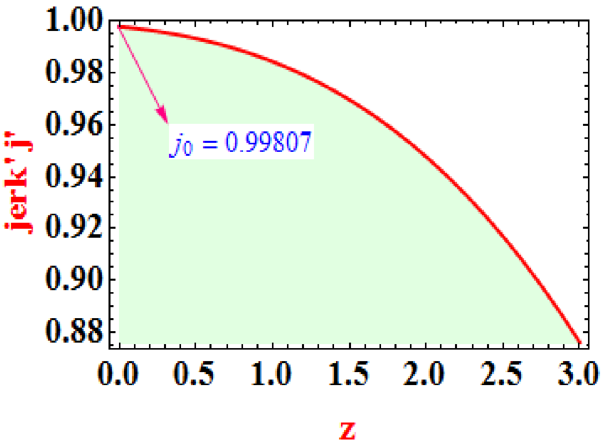

Fig. 6. Plot of jerk j versus z.

$$3\frac{\dot{a}^2}{a^2} = 8\pi G\ \rho_{eff},\ \ \rho_{eff} = \rho_m + \Lambda/8\pi G \tag{20}$$

Solving equations (19) and (20), we obtain the following expressions for ρ_{eff} and p_{eff}

$$\rho_{eff} = (\rho_c)_0 \left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right) \tag{21}$$

$$p_{eff} = \omega\ \rho_{eff} - (1+\omega)(\rho_c)_0\ (\Omega_\Lambda)_0 = (\rho_c)_0\ \left(\omega\left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right) - (1+\omega)(\Omega_\Lambda)_0\right) \tag{22}$$

We also define effective equation of state parameter ω_{eff} as follows

$$\omega_{eff} = p_{eff}/\rho_{eff} = \frac{\left(\omega\left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right) - (1+\omega)(\Omega_\Lambda)_0\right)}{\left((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\right)} \tag{23}$$

From equations (21) - (23), we observe the following facts:

- The present value of effective pressure and EoS parameter ω_{eff} is as below

  $$p_{eff} = -(\rho_c)_0(\Omega_\Lambda)_0,\ \ (\omega_{eff})_0 = -(\Omega_\Lambda)_0 \sim -0.7.$$

  Fortunately, these confirm that the universe is accelerating at present.
- For dust filled universe (ω=0), ω_{eff} increases over red shift and tend to zero. This means that the universe has interred into accelerating phase. This behavior is graphed into left panel of Fig. 6.

- At radiation era, when universe was highly warm and turbulent, $p_m = \rho_m/3\ \omega = 1/3$. This was the period when red shift $z \geq 1100$. During this

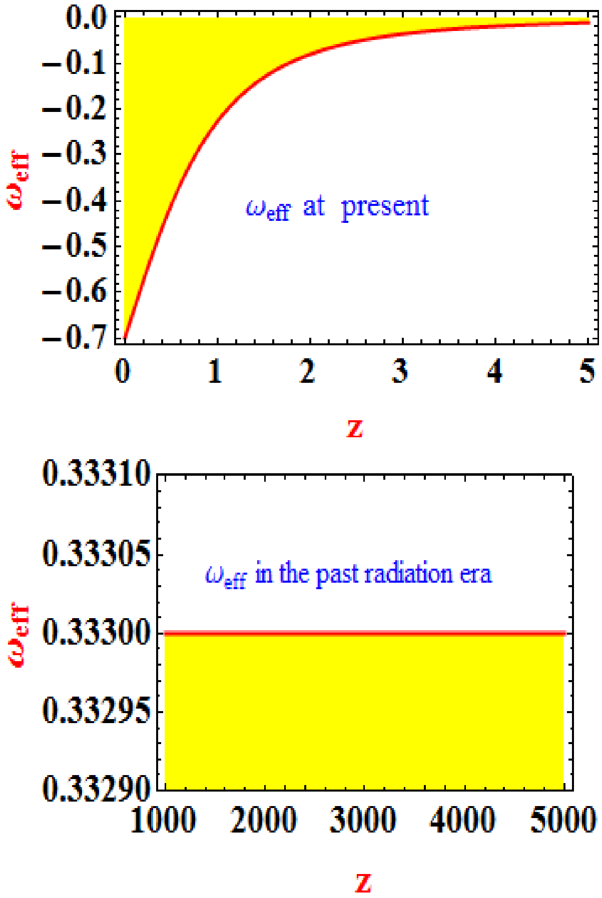

Fig. 7. Plot of ω_{eff} versus z.

span of time, ω_{eff} is as follow

$$\omega_{eff} = 0.333 - \frac{1.333(\Omega_\Lambda)_0}{\big((\Omega_m)_0(1+z)^{3(1+\omega)} + (\Omega_\Lambda)_0\big)}$$

Thus $\omega_{eff} \simeq \omega = 1/3$ and $p_{eff} \simeq 1/3\ \rho_{eff}$, when $z \geq 1100$. This behaviour of ω_{eff} is shown in the right panel of Fig. 6.

- It is concluded that inclusion of Λ in the energy content of the universe does not change the physics of early universe but it is required to explain the present acceleration.

5. Conclusion

In this paper, we have investigated an exact solution of Einstein's field equation in FRW space-time. We have examined the model parameters of derived model with its observed values through statistical χ^2 test. The summary of numerical analysis is given in table 1.

Model parameters	H(z) data	Pantheon data
H_0	68.13 ± 1.17	71.47 ± 0.53
$(\Omega_m)_0$	0.27 ± 0.005	0.276 ± 0.006
$(\Omega_\Lambda)_0$	0.718 ± 0.015	0.74 ± 0.01

The main features of derived model are as follows:

i) We have derived an exact and new solution of Einstein's field equations rather than an adhoc assumptions as taken in [24,25,26]

ii) The derived universe represents a model of transitioning universe which was in decelerated expanding phase for $z_t > 0.723$. The current universe is expanded with acceleration and the present value of deceleration parameter is -0.60. The value of deceleration parameter for pure dust model is -0.55[8,27].

iii) The present age of the universe in the derived model is 13.89 ± 0.017.

iv) We have obtained the effective energy density and pressure of the derived universe which describes the dynamics of universe from its beginning to present time.

v) It is important to note that at $t = -\frac{c_0}{\Lambda_0(1+\omega)}$, the scalar factor (a) and volume scalar $(V = a^3)$ vanish. Therefore, the universe in derived model has big bang singularity at $t = -\frac{c_0}{\Lambda_0(1+\omega)}$.

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